

Ingredients for BRST invariance**Exercise 1:**

(a) Verify the Jacobi identity [the Einstein summation convention is assumed]:

$$f^{abe} f^{cde} + f^{cae} f^{bde} + f^{bce} f^{ade} = 0 .$$

(b) Let $\mathcal{D}_\mu^{ab}(x) \equiv \delta^{ab} \partial_\mu + g f^{acb} A_\mu^c(x)$ and let $\tau(x), \sigma(y)$ be arbitrary “test functions”. Verify that

$$\int d^4y \left[\frac{\delta \mathcal{D}_\mu^{be}(x)}{\delta A_\nu^d(y)} \mathcal{D}_\nu^{dc}(y) - \frac{\delta \mathcal{D}_\mu^{bc}(y)}{\delta A_\nu^d(x)} \mathcal{D}_\nu^{de}(x) \right] \tau(x) \sigma(y) = g f^{cea} \mathcal{D}_\mu^{ba}(x) \tau(x) \sigma(x) .$$

Exercise 2: Let

$$\Delta c^a \equiv \frac{g}{2} f^{abc} c^b c^c ,$$

where the c 's are Grassmann variables. Verify that $\Delta^2 c^a = 0$.