

Ingredients for anomalies

Exercise 1: Let $\bar{\psi}_a$, ψ_a , $a = 1, \dots, N$, be Grassmann variables, and $\bar{\psi}'_a \equiv \bar{\psi}_a + \bar{\psi}_b \bar{M}_{ba}$, $\psi'_a \equiv \psi_a + M_{ab} \psi_b$, where a summation over b is implied. Verify the validity of these relations to first order in M and \bar{M} :

$$(a) \quad \left\{ \prod_a d\bar{\psi}'_a \right\} = \left\{ \prod_a d\bar{\psi}_a \right\} \{1 - \text{tr} [\bar{M}]\},$$
$$(b) \quad \left\{ \prod_a d\psi'_a \right\} = \left\{ \prod_a d\psi_a \right\} \{1 - \text{tr} [M]\}.$$

Exercise 2: Show that for $\theta = n\pi$, $n \in \mathbb{Z}$, $\exp(i\theta\gamma_5) = (-1)^n$.