Quantum field theory II	**Exercise sheet 10**

Symmetries and Ward identities

Exercise 1: Let us consider (Euclidean) complex scalar field theory, i.e.

$$\mathcal{L}_E = \partial_\mu \phi^* \partial_\mu \phi - V(\phi^* \phi) \; .$$

This theory is invariant in the global transformation $\phi \to e^{i\omega}\phi$, $\phi^* \to e^{-i\omega}\phi^*$. Derive the Noether current, through

(a) the definition $\frac{\delta S_E}{\delta \omega(x)} \equiv -\partial_\mu j_\mu(x)$.

(b) a comparison with Exercise 2.2 of QFTI. [Hint: Write $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$.]

Exercise 2: Consider a "linearly realized" symmetry

$$\delta\phi^a(x) = \Phi^a_i(x)\delta\omega^i \equiv T^{ab}_i \phi^b(x)\delta\omega^i \; ,$$

where the T^{ab}_i are constants, and $\partial S_E[\phi^a]/\partial\omega^i = 0$. Show that

(a) $\int \mathrm{d}^4x \, T^{ab}_i \phi^b(x) \frac{\delta S_E[\phi]}{\delta\phi^a(x)} = 0$,

(b) $\int \mathrm{d}^4x \, T^{ab}_i J^a(x) \frac{\delta Z[J]}{\delta J^b(x)} = 0$,

(c) $\int \mathrm{d}^4x \, T^{ab}_i \varphi^b(x) \frac{\delta \Gamma[\varphi]}{\delta\varphi^a(x)} = 0$.

Here

$$Z[J] \;\equiv\; \int \mathcal{D}\phi \, \exp\Big\{ -S_E[\phi] + \int_x J(x)\phi(x) \Big\} \;\equiv\; \exp\{W[J]\} \; ,$$

$$\Gamma[\varphi] \;\equiv\; W[J] - \int_x \varphi(x)J(x) \quad \text{with} \quad \varphi(x) \;\equiv\; \delta W[J]/\delta J(x) \; .$$

[A comparison of points (a) and (c) implies that the "effective action", $\Gamma[\varphi]$, displays the same symmetries as the classical one, $S_E[\phi]$.]