

1-loop gauge field self-energy in Feynman background field gauge

Exercise 1: Let a self-energy read

$$\begin{aligned}\Pi_{\mu\nu}(Q) &= \frac{g^2 N_c}{2} \int \frac{d^d P}{(2\pi)^d} \frac{1}{P^2(P+Q)^2} \left[4(d-2)P_\mu P_\nu + 2(d-2)(P_\mu Q_\nu + P_\nu Q_\mu) \right. \\ &\quad \left. + (d-10)Q_\mu Q_\nu + 8\delta_{\mu\nu}Q^2 + 2(2-d)\delta_{\mu\nu}(P+Q)^2 \right].\end{aligned}$$

Verify that the self-energy is transverse, i.e. $Q_\mu Q_\nu \Pi_{\mu\nu}(Q) = 0$.

Exercise 2: After inserting the result from Exercise 13.2 of QFTI, show that ($d \equiv 4-2\epsilon$)

$$\int \frac{d^d P}{(2\pi)^d} \frac{1}{P^2(P+Q)^2} = \frac{\mu^{-2\epsilon}}{(4\pi)^2} \left[\frac{1}{\epsilon} + \ln \frac{\mu^2}{Q^2} + \mathcal{O}(1) \right].$$