Quantum field theory II **Exercise sheet 8**

Background field gauge, determination of renormalization constants

Exercise 1: Starting with the definitions $[T^a, T^b] = if^{abc}T^c$, $\mathrm{tr}\,[T^aT^b] = \delta^{ab}/2$, $D_\mu(A) = \mathbb{1}_{N_\mathrm{c}\times N_\mathrm{c}}\,\partial_\mu - igT^aA^a_\mu$, $\mathcal{D}^{ab}_\mu(A) = \delta^{ab}\partial_\mu + gf^{acb}A^c_\mu$, show that

$$\int \mathrm{d}^4x\; \bar{c}^a\Big\{-\mathcal{D}^{ac}_\mu(B)\mathcal{D}^{cb}_\mu(A+B)\Big\}c^b = \int \mathrm{d}^4x\; 2\,\mathrm{tr}\,\Big\{[D_\mu(B),\bar{c}][D_\mu(A+B),c]\Big\}\;,$$

where $c = c^aT^a$ and $\bar{c} = \bar{c}^aT^a$.

Exercise 2: Let us inspect $\Gamma[B]$ in a general gauge (not necessarily the background field gauge). Let us assume that the coefficients of the terms B^2_B, B^3_B and B^4_B have already been computed. In which ways could we extract the renormalization constant Z_g? [Hint: When expressed in terms of renormalized quantities, $\Gamma[B]$ must be finite.]