Quantum field theory II	**Exercise sheet 7**

Non-Abelian propagator and vertices

Exercise 1: If the quadratic part of an action can be written as

$$S_E^{(2)} = \int_{P,Q} \bar{\delta}(P+Q)\,\frac{1}{2}\tilde{A}^a_\mu(P)\tilde{A}^a_\nu(Q)\tilde{\Delta}_{\mu\nu}(P)\;,$$

then the propagator reads

$$\langle \tilde{A}^a_\mu(P)\tilde{A}^b_\nu(Q)\rangle = \delta^{ab}\,\bar{\delta}(P+Q)\,\tilde{\Delta}^{-1}_{\mu\nu}(P)\;.$$

Determine $\tilde{\Delta}^{-1}_{\mu\nu}(P)$ for the case

$$\tilde{\Delta}_{\mu\nu}(P) = P^2\delta_{\mu\nu} - \left(1-\frac{1}{\xi}\right)P_\mu P_\nu\;.$$

What would happen with the derivation if the gauge fixing term were left out?

Exercise 2: The interaction terms of pure Yang-Mills theory are written as

$$\begin{aligned} S_E^{(3)} + S_E^{(4)} \;&=\; \int_{P,Q,R} \bar{\delta}(P+Q+R)\,\frac{1}{3!}\tilde{A}^a_\mu(P)\tilde{A}^b_\nu(Q)\tilde{A}^c_\rho(R)V^{abc}_{\mu\nu\rho}(P,Q,R) \\ &+\; \int_{P,Q,R,S} \bar{\delta}(P+Q+R+S)\,\frac{1}{4!}\tilde{A}^a_\mu(P)\tilde{A}^b_\nu(Q)\tilde{A}^c_\rho(R)\tilde{A}^d_\sigma(S)V^{abcd}_{\mu\nu\rho\sigma}(P,Q,R,S)\;. \end{aligned}$$

Show that

$$\begin{aligned} V^{abc}_{\mu\nu\rho}(P,Q,R) \;&=\; igf^{abc}\Big[\delta_{\mu\rho}(P_\nu - R_\nu) + \delta_{\rho\nu}(R_\mu - Q_\mu) + \delta_{\nu\mu}(Q_\rho - P_\rho)\Big]\;, \\ V^{abcd}_{\mu\nu\rho\sigma}(P,Q,R,S) \;&=\; g^2\Big[f^{eab}f^{ecd}\Big(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho}\Big) + f^{eac}f^{ebd}\Big(\delta_{\mu\nu}\delta_{\rho\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho}\Big) \\ &\quad + f^{ead}f^{ebc}\Big(\delta_{\mu\nu}\delta_{\rho\sigma} - \delta_{\mu\rho}\delta_{\nu\sigma}\Big)\Big]\;. \end{aligned}$$