Quantum field theory II	**Exercise sheet 6**

Topological charge density, Wilson line

Exercise 1: We consider $C(x) \equiv \mathrm{tr}\,[F_{\mu\nu}(x)F_{\rho\sigma}(x)]\epsilon^{\mu\nu\rho\sigma}$, where $\epsilon^{\mu\nu\rho\sigma}$ is the totally antisymmetric 4-dimensional Levi-Civita tensor.

(a) Show that $C(x)$ is gauge invariant.

(b) Show that $C(x)$ is invariant in Lorentz transformations, if $\det\Lambda = +1$.

(c) Verify that $C(x)$ can be expressed as

$$\begin{aligned} C(x) &= \partial_\mu K^\mu(x) \; , \\ K^\mu(x) &= 2\epsilon^{\mu\nu\rho\sigma}\Big(A^a_\nu \partial_\rho A^a_\sigma + \frac{g}{3} f^{abc} A^a_\nu A^b_\rho A^c_\sigma\Big) \; . \end{aligned}$$

[Because of this fact, $C(x)$ integrates to a boundary term in the action, and thus plays no role in the classical equations of motion.]

Exercise 2: A "Wilson line" $W(x,x_0)$ in the μ direction satisfies the equations

$$\begin{aligned} D_\mu(x) W(x,x_0) &= 0 \; , \\ W(x_0,x_0) &= \mathbb{1}_{N_c\times N_c} \; . \end{aligned}$$

(a) Write down a formal solution for $W(x,x_0)$.

(b) Verify that $W(x,x_0)$ behaves in gauge transformations as

$$W(x,x_0) \to U(x) W(x,x_0) U^\dagger(x_0) \; .$$

[Because of this fact, a Wilson line permits for a gauge invariant comparison of quantities defined at x und x_0, and is therefore called a "parallel transporter".]