Quantum field theory II	**Exercise sheet 4**

Charge conjugation and parity

Exercise 1: In the so-called standard representation we write

$$\gamma^0 = \begin{pmatrix} \mathbb{1}_{2\times 2} & 0 \\ 0 & -\mathbb{1}_{2\times 2} \end{pmatrix} , \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} ,$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} .$$

The Euclidean Dirac matrices are defined as $\gamma_0 \equiv \gamma_0^E \equiv \gamma^0$, $\gamma_i \equiv \gamma_i^E \equiv -i\gamma^i$. Show that $C \equiv \gamma_0\gamma_2$ satisfies

$$C\gamma_\mu C^{-1} = -\gamma_\mu^T ,$$

and has the properties $C^\dagger = C^{-1} = C^T = -C$.

Exercise 2: We consider the action

$$S_E \equiv \int \mathrm{d}\tau \mathrm{d}^3\vec{x}\, \bar{\psi}[\gamma_\mu\partial_\mu + m]\frac{1}{2}(1-\gamma_5)\psi .$$

How does S_E behave in the discrete transformations P and C?