Quantum field theory II	**Exercise sheet 3**

Wick contractions with Grassmann fields, properties of γ_5

Exercise 1: Let

$$\langle ... \rangle \equiv \frac{\int \{\prod_i \mathrm{d}c_i^* \mathrm{d}c_i\} (...) \exp(-\sum_{p,q} c_p^* A_{pq} c_q)}{\int \{\prod_i \mathrm{d}c_i^* \mathrm{d}c_i\} \exp(-\sum_{p,q} c_p^* A_{pq} c_q)} ,$$

where c^* and c are Grassmann variables. What do you find for the correlators

$$\langle c_k c_l c_m c_n \rangle , \quad \langle c_k c_l c_m c_n^* \rangle , \quad \langle c_k c_l c_m^* c_n^* \rangle ?$$

Exercise 2: Let us consider Euclidean versions of the Dirac metrices, as defined in the script. Verify the following properties of $\gamma_5 \equiv \gamma_0\gamma_1\gamma_2\gamma_3$:

(a) $\mathrm{tr}\,[\gamma_5] = 0$.

(b) $\mathrm{tr}\,[\text{uneven number of } \gamma\text{'s}] = 0$, and consequently $\mathrm{tr}\,[\gamma_5\gamma_\mu] = \mathrm{tr}\,[\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho] = 0$.

(c) $\mathrm{tr}\,[\gamma_5\gamma_\mu\gamma_\nu] = 0$.

(d) $\mathrm{tr}\,[\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma] = N\epsilon_{\mu\nu\rho\sigma}$, with $N = \mathrm{tr}\,[\gamma_0^2]$ and

$$\epsilon_{\mu\nu\rho\sigma} \equiv \begin{cases} +1, \text{ if } \mu\nu\rho\sigma \text{ is an even permutation of 0123,} \\ -1, \text{ if } \mu\nu\rho\sigma \text{ is an odd permutation of 0123,} \\ 0, \text{ otherwise.} \end{cases}$$

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