Quantum field theory II **Exercise sheet 2**

Grassmann integrals

Exercise 1: Let M be a 2×2 matrix, $c^{*T} \equiv (c_1^* \; c_2^*)$, $c \equiv (c_1 \; c_2)^T$, and $S_E \equiv c^{*T} M c$. Compute the Grassmann integral

$$I(M) \equiv \int \mathrm{d}c_1^* \mathrm{d}c_1 \mathrm{d}c_2^* \mathrm{d}c_2 \, \exp(-S_E) \, .$$

Exercise 2: With S_E like in Exercise 1, compute the correlators

(a) $$\langle c_1 c_1^* \rangle \equiv \frac{\int \mathrm{d}c_1^* \mathrm{d}c_1 \mathrm{d}c_2^* \mathrm{d}c_2 \, c_1 c_1^* \, \exp(-S_E)}{\int \mathrm{d}c_1^* \mathrm{d}c_1 \mathrm{d}c_2^* \mathrm{d}c_2 \, \exp(-S_E)} \, ,$$

(b) $$\langle c_1 c_2^* \rangle \, ,$$

(c) $$\langle c_2 c_1^* \rangle \, ,$$

(d) $$\langle c_2 c_2^* \rangle \, .$$

Can you find a compact way to express the general correlator $\langle c_i c_j^* \rangle$?