Quantum field theory II	**Exercise sheet 1**

Fock space and canonical anticommutators for fermions

Exercise 1: We define the Fock space for fermions just like for bosons, i.e.

$$|\vec{k}_1, ..., \vec{k}_n\rangle = \hat{a}^\dagger_{\vec{k}_1} ... \hat{a}^\dagger_{\vec{k}_n} |0\rangle \ ,$$

where the creation operators are now anticommuting. Verify the Pauli principle, i.e.

$$|\vec{k}_1, ..., \vec{k}_i, ..., \vec{k}_j, ..., \vec{k}_n\rangle = -|\vec{k}_1, ..., \vec{k}_j, ..., \vec{k}_i, ..., \vec{k}_n\rangle \ .$$

Exercise 2: Starting with the anticommutators for the fermionic annihilation and creation operators, verify the validity of

$$\left\{\hat{\psi}_\alpha(x^0, \vec{x}), i\hat{\psi}^\dagger_\beta(x^0, \vec{y})\right\} = i\delta^{(3)}(\vec{x} - \vec{y})\delta_{\alpha\beta} \ , \quad \alpha, \beta = 1, ..., 4 \ .$$

Here $\hat{\psi}^\dagger_\beta \ \equiv \ \hat{\bar{\psi}}_\alpha \gamma^0_{\alpha\beta}$. [Hint: Make use of the completeness relation for the spinors $u(\vec{p}, s)$, $v(\vec{p}, s)$ as well as the substitution $\vec{p} \to -\vec{p}$.]