

Renormalization constants and the renormalization group

Exercise 1: Show that the renormalization constant δZ_ϕ in scalar field theory can be at most of order $\mathcal{O}(\lambda_R^2)$ for $\lambda_R \ll 1$.

Exercise 2: Show that the solution of the renormalization group equation

$$\mu \frac{d}{d\mu} \lambda_R = \frac{3}{(4\pi)^2} \lambda_R^2$$

can be written as

$$\lambda_R(\mu) = \frac{(4\pi)^2}{3 \ln(\mu_0/\mu)},$$

where μ_0 is an integration constant.