Quantum field theory I	**Exercise sheet 11**

Schwinger-Dyson equations

Exercise 1: Let us define $\mathcal{L}_E \equiv \frac{1}{2}\partial_\mu\phi\,\partial_\mu\phi + \frac{1}{2}m^2\phi^2 + \frac{g}{3!}\phi^3 + \frac{\lambda}{4!}\phi^4$ and $S_E \equiv \int \mathrm{d}^4x\,\mathcal{L}_E$. Verify the validity of the relation

$$\frac{\delta S_E}{\delta\phi(x)} = \left[-\partial_\mu^2 + m^2\right]\phi(x) + \frac{g}{2!}\phi^2(x) + \frac{\lambda}{3!}\phi^3(x)\ .$$

Exercise 2: In the script we have defined the quantities $Z[J]$, $W[J] \equiv \ln Z[J]$, and $\Gamma[\varphi] \equiv W[J] - \int \mathrm{d}^4x\,\varphi(x)J(x)$, where $\varphi(x) \equiv \delta W[J]/\delta J(x)$. Starting from the Schwinger-Dyson equation

$$0 = \left[-\mathcal{L}'_E\left(\frac{\delta}{\delta J(x)}\right) + J(x)\right]Z[J]\ ,$$

verify the validity of the following relations:

$$\text{(a)}\qquad \mathcal{L}'_E\left(\frac{\delta W[J]}{\delta J(x)} + \frac{\delta}{\delta J(x)}\right) = J(x)\ ,$$

$$\text{(b)}\qquad \mathcal{L}'_E\left(\varphi(x) + \int \mathrm{d}^4y\,D[\varphi](x,y)\frac{\delta}{\delta\varphi(y)}\right) = -\frac{\delta\Gamma[\varphi]}{\delta\varphi(x)}\ ,$$

where $D[\varphi](x,y) \equiv \delta^2 W[J]/\delta J(x)\delta J(y)$.