Quantum field theory I **Exercise sheet 10**

Generating functional, finite-volume momentum space

Exercise 1: If an integration measure is defined as

$$\int \mathrm{d}\vec{v} \;\equiv\; \int_{-\infty}^{\infty} \left[\prod_i \frac{\mathrm{d}v_i}{\sqrt{2\pi}} \right] ,$$

what do you obtain for $e^{W(0)}$ from Exercise 9.2? [Answer: $e^{W(0)} = (\det A)^{-1/2}$].

Exercise 2: Let us consider the four-volume $V = L_0 L_1 L_2 L_3$ with periodic boundary conditions in all directions.

(a) Show that in the infinite-volume limit $\lim_{V\to\infty} \frac{1}{V} \sum_P \tilde{f}(P) = \int \frac{\mathrm{d}^4 P}{(2\pi)^4} \tilde{f}(P)$.

(b) A normalized δ function, $\bar{\delta}(P)$, is defined through the condition

$$\int_P \bar{\delta}(P - Q) \tilde{f}(P) = \tilde{f}(Q) \; ,$$

with $\int_P = \frac{1}{V} \sum_P$ in finite volume, and $\int_P = \int \frac{\mathrm{d}^4 P}{(2\pi)^4}$ in infinite volume. Write an explicit expression for $\bar{\delta}(P)$ in both cases.

(c) Verify the validity of $\int_V \mathrm{d}^4 x \, e^{iP\cdot x} = \bar{\delta}(P)$ both for finite and infinite V.