Quantum field theory I	**Exercise sheet 9**

Euclidean correlators, Gaussian integrals

Exercise 1: We define

$$G_E^{(n)}(\tau_1, ..., \tau_n) \equiv \langle 0|T\{\hat{x}_H(\tau_1) \dots \hat{x}_H(\tau_n)\}|0\rangle \ ,$$
$$G_\beta^{(n)}(\tau_1, ..., \tau_n) \equiv \frac{\mathrm{Sp}\,[e^{-\beta\hat{H}}T\{\hat{x}_H(\tau_1) \dots \hat{x}_H(\tau_n)\}]}{\mathrm{Sp}\,[e^{-\beta\hat{H}}]} \ ,$$

with $0 \le \tau_1, ..., \tau_n \le \beta$. Verify the validity of $\lim_{\beta\to\infty} G_\beta^{(n)}(\tau_1, ..., \tau_n) = G_E^{(n)}(\tau_1, ..., \tau_n)$.

Exercise 2: Starting with the definition and result

$$e^{W(J)} \equiv \int \mathrm{d}\vec{v}\, \exp\left[-\frac{1}{2}v_i A_{ij} v_j + J_i v_i\right] = e^{W(0)} \exp\left[\frac{1}{2}J_i A^{-1}_{ij} J_j\right] \ ,$$

compute the expectation value

$$\langle v_m v_n v_o v_p \rangle_0 \equiv \frac{\int \mathrm{d}\vec{v}\; v_m v_n v_o v_p\; \exp\left[-\frac{1}{2}v_i A_{ij} v_j\right]}{\int \mathrm{d}\vec{v}\; \exp\left[-\frac{1}{2}v_i A_{ij} v_j\right]} \ ,$$

in terms of A^{-1}.