

**Wick's theorem and contractions**

**Exercise 1:** Verify the validity of Wick's theorem up to 3rd order:

$$\begin{aligned}
 T\{\hat{\phi}_I(x_1)\hat{\phi}_I(x_2)\hat{\phi}_I(x_3)\} &= : \hat{\phi}_I(x_1)\hat{\phi}_I(x_2)\hat{\phi}_I(x_3) : \\
 &+ : \hat{\phi}_I(x_1) : \langle 0|T\{\hat{\phi}_I(x_2)\hat{\phi}_I(x_3)\}|0\rangle \\
 &+ : \hat{\phi}_I(x_2) : \langle 0|T\{\hat{\phi}_I(x_3)\hat{\phi}_I(x_1)\}|0\rangle \\
 &+ : \hat{\phi}_I(x_3) : \langle 0|T\{\hat{\phi}_I(x_1)\hat{\phi}_I(x_2)\}|0\rangle .
 \end{aligned}$$

**Exercise 2:** Let  $\mathcal{L}_{\text{int}} \equiv -\frac{1}{4!}\lambda\phi^4$  and therefore, like in the script,

$$\hat{H}_I(t) = \int d^3\vec{x} \frac{1}{4!}\lambda\hat{\phi}_I^4(t, \vec{x}) .$$

Determine the amputated Green's function  $\tilde{A}_{T,c}^{(4)}(p_1, \dots, p_4)$  up to first order in  $\lambda$ .