

Green's functions, normal ordering**Exercise 1:**

(a) Consider the definition

$$(2\pi)^4 \delta^{(4)}(p_1 + \dots + p_n) \tilde{G}_{T,c}^{(n)}(p_1, \dots, p_n) \\ \equiv \int d^4 x_1 \dots \int d^4 x_n G_{T,c}^{(n)}(x_1, \dots, x_n) e^{i(p_1 \cdot x_1 + \dots + p_n \cdot x_n)} .$$

Which property of $G_{T,c}^{(n)}$ guarantees the existence of the Dirac- δ in front of $\tilde{G}_{T,c}^{(n)}$?

(b) The Green's function $G_T^{(n)}$ can also contain disconnected parts, i.e. $G_T^{(n)} = \dots + G_T^{(m_1)} G_T^{(m_2)}$, with $m_1 + m_2 = n$. How do such parts manifest themselves in $\tilde{G}_T^{(n)}$?

Exercise 2:

(a) Verify explicitly the validity of

$$: \hat{\phi}_I(x_1) \hat{\phi}_I(x_2) : = : \hat{\phi}_I(x_2) \hat{\phi}_I(x_1) : .$$

(b) Generalize the result to show that field operators can be commuted with each other within normal ordering, i.e.

$$: \dots \hat{\phi}_I(x_i) \dots \hat{\phi}_I(x_j) \dots : = : \dots \hat{\phi}_I(x_j) \dots \hat{\phi}_I(x_i) \dots : .$$