Quantum field theory I	**Exercise sheet 6**

Time evolution operator

Exercise 1: Let $\hat{U}_I(t, t_0)$ be the time evolution operator, as defined in the script.

(a) Express $\hat{U}_I(t, t_0)$ with the help of $e^{i\hat{H}t}$, $e^{i\hat{H}t_0}$, $e^{i\hat{H}_0 t}$ and $e^{i\hat{H}_0 t_0}$.

(b) Verify the property $\hat{U}_I^\dagger(t_1, t_2) = \hat{U}_I(t_2, t_1)$.

(c) Verify the property $\hat{U}_I(t_1, t_2)\hat{U}_I(t_2, t_3) = \hat{U}_I(t_1, t_3)$ for $t_1 \geq t_2 \geq t_3$.

(d) Verify the property $\hat{U}_I(t_1, t_3)\hat{U}_I^\dagger(t_2, t_3) = \hat{U}_I(t_1, t_2)$ for $t_1 \geq t_2 \geq t_3$.

Exercise 2: Let $\hat{S}$ be the scattering matrix, as defined in the script. Show that $\hat{S}$ is unitary:

$$\hat{S}^\dagger \hat{S} = \hat{S}\, \hat{S}^\dagger = \mathbb{1} \ .$$