Quantum field theory I	**Exercise sheet 5**

Properties of free propagators

Exercise 1: Verify the following properties:

$$\text{(a)} \qquad \int_{-\infty}^{\infty} \frac{\mathrm{d}\tilde{k}}{2\pi} \frac{e^{i\tilde{k}\tau}}{\tilde{k}^2 + E^2} = \frac{1}{2E}\Big[\theta(-\tau)e^{E\tau} + \theta(\tau)e^{-E\tau}\Big] \, ,$$

$$\text{(b)} \qquad \lim_{\epsilon\to 0^+} \int_{-\infty}^{\infty} \frac{\mathrm{d}k}{2\pi} \frac{e^{ikt}}{k^2 - E^2 + i\epsilon} = \frac{-i}{2E}\Big[\theta(-t)e^{iEt} + \theta(t)e^{-iEt}\Big] \, .$$

[Hint: make use of the residue theorem.]

Exercise 2: In Fourier space, the Schwinger propagator reads

$$\tilde{G}_E(\tilde{p}^0; \vec{p}\,) = \frac{1}{(\tilde{p}^0)^2 + E_{\vec{p}}^2} \, ,$$

and the spectral function is

$$\tilde{\rho}(p^0; \vec{p}\,) = \frac{\pi}{2E_{\vec{p}}}\Big[\delta(p^0 - E_{\vec{p}}) - \delta(p^0 + E_{\vec{p}})\Big] \, .$$

Show that the spectral function can be obtained es

$$\tilde{\rho}(p^0, \vec{p}\,) = \mathrm{Im}\, \tilde{G}_E(\tilde{p}^0 \to -i(p^0 + i0^+); \vec{p}\,) \, .$$

[Hint: recall different representations of the Dirac-δ.]