

**Fock space of a free scalar field**

**Exercise 1:** We consider the Fock space of a free scalar field, and define

$$\begin{aligned}
 |\vec{k}_1, \vec{k}_2, \dots, \vec{k}_n\rangle &\equiv \hat{a}_{\vec{k}_1}^\dagger \hat{a}_{\vec{k}_2}^\dagger \dots \hat{a}_{\vec{k}_n}^\dagger |0\rangle, \\
 \hat{N} &\equiv \int d^3\vec{p} \hat{a}_{\vec{p}}^\dagger \hat{a}_{\vec{p}}, \\
 :\hat{H}: &\equiv \int d^3\vec{p} E_{\vec{p}} \hat{a}_{\vec{p}}^\dagger \hat{a}_{\vec{p}}.
 \end{aligned}$$

What are  $\hat{N}|\vec{k}_1, \vec{k}_2, \dots, \vec{k}_n\rangle$  and  $:\hat{H}:|\vec{k}_1, \vec{k}_2, \dots, \vec{k}_n\rangle$ ?

**Exercise 2:**

- (a) Show that  $|\vec{k}_1, \vec{k}_2\rangle = |\vec{k}_2, \vec{k}_1\rangle$ , i.e., that the particles are bosons.
- (b) Determine the amplitude  $\langle \vec{k}'_1, \vec{k}'_2 | \vec{k}_1, \vec{k}_2 \rangle$ .