Quantum field theory I	**Exercise sheet 2**

Energy-momentum tensor and particle number current

Exercise 1: Consider the Lorentz transformation $x^\mu \to x^{\mu\prime} \equiv \Lambda^\mu{}_\nu x^\nu$, where $\eta^{\alpha\beta}\Lambda^\mu{}_\alpha \Lambda^\nu{}_\beta = \eta^{\mu\nu}$. We assume that the field ϕ is invariant in this substitution, $\phi(x) \to \phi'(x') \equiv \phi(x)$.

(a) Show that the action $S = \int \mathrm{d}^4x\, \mathcal{L}$, where $\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)$, is Lorentz invariant.

(b) Show that an infinitesimal $\Lambda^\mu{}_\alpha$ can be written as $\Lambda^\mu{}_\alpha = \delta^\mu{}_\alpha + \epsilon^\mu{}_\alpha$, where the generators are antisymmetric, $\epsilon^{\mu\nu} = -\epsilon^{\nu\mu}$.

(c) What are the corresponding Noether currents?

Exercise 2: We consider now

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi_1\partial_\mu\phi_1 + \frac{1}{2}\partial^\mu\phi_2\partial_\mu\phi_2 - V(\phi_1^2 + \phi_2^2) \ , \tag{1}$$

and carry out the substitution

$$\delta\phi_1 \equiv -\phi_2\,\delta\omega \ , \quad \delta\phi_2 \equiv \phi_1\,\delta\omega \ . \tag{2}$$

Verify that this is an invariance. What is the Noether current?