Quantum field theory I	**Exercise sheet 1**

Noether theorem

Exercise 1: Consider a general Lagrangian density $\mathcal{L}(\phi, \partial_\mu \phi)$, which contains no explicit dependence on x.

(a) Show that $\mathcal{L}$ is invariant in the substitution

$$x^\mu \to x^{\mu\prime} = x^\mu + \omega^\mu \ , \quad \phi(x) \to \phi'(x') = \phi(x) \ . \tag{1}$$

(b) What are the corresponding Noether currents j^μ_ν? [Hint: $X^\mu_i \equiv \delta^\mu{}_\nu$.]

(c) We denote in this case $j^\mu_\nu \equiv T^\mu{}_\nu$. Show that $T^{00} = \mathcal{H}$ (i.e. the energy density equals the Hamiltonian density).

Exercise 2: Let us assume that $\partial_\mu j^\mu_i = 0$. Under which conditions is the "charge" $Q_i \equiv \int_V \mathrm{d}^3\vec{x}\, j^0_i$ conserved?