exercises for mechanics I	**sheet 4**

[tutorials 24.3. and 25.3., hand-in 4.4., solutions please in English]

Exercise 1 (Foucault pendulum): Consider a point mass attached to a pendulum of length l, undergoing 3-dimensional movement in the gravitational field $\vec{g} = -g\vec{e}_z$. The coordinates of the rest position are $x = y = z = 0$.

(a) Show that to first order in deviations from the rest position the equations of motion take the form $\ddot{x} = -gx/l$, $\ddot{y} = -gy/l$. Solve these for the initial conditions $x(0) = r_0$, $y(0) = 0$, $\dot{x}(0) = 0$, $\dot{y}(0) = v_0$, and sketch the trajectory. (3 points)

(b) Now we include the rotation of the earth and the Coriolis force. At latitude θ, the angular velocity of earth rotation takes the form $\vec{\omega} = (-\omega\cos\theta, 0, \omega\sin\theta)$ in the above coordinate system. Show that if we omit Ω^2 compared with g/l, then the solution in the presence of the Coriolis force can be expressed as

$$\begin{pmatrix} X(t) \\ Y(t) \end{pmatrix} = \begin{pmatrix} \cos(\Omega t) & \sin(\Omega t) \\ -\sin(\Omega t) & \cos(\Omega t) \end{pmatrix} \begin{pmatrix} x(t) \\ y(t) \end{pmatrix},$$

where $(x(t), y(t))$ is the solution from point (a) and $\Omega \propto \omega$ is a constant to be determined. Sketch the trajectory. (3 points)

Exercise 2: On a town square at latitude θ stands a tower of height H. The square is defined as the (x, y) plane, the tower as the z axis. Because of earth rotation we are faced with a rotating system. Determine how far a freely falling object (initial velocity zero) lands from the bottom of the tower. (6 points) [Hint: terms of order ω^2 can be omitted. Answer: $\frac{2H\omega}{3}\sqrt{\frac{2H}{g}}\cos\theta$.]

Exercise 3: Consider an electron (e^-) and a positron (e^+). At time $t = 0$ their relative separation is $\vec{r}(0) = r_0\,\vec{e}_x$ with $r_0 = 10^{-10}$ m, and their relative velocity $\vec{v}(0) = v_0\,\vec{e}_y$ with $v_0 = \alpha c$, where $\alpha = 1/137$ and c is the speed of light.

(a) Determine the reduced mass, the inner kinetic energy, the potential energy, the total inner energy, and the angular momentum $\vec{L}$. (4 points)

(b) The system described, known as positronium, is a bound state. Why? Would this be the case if v_0 were three times as large? (2 points)

Exercise 4: A mass μ moves with energy E and angular momentum $\ell = |\vec{L}|$ in the potential well $U(r) = -\alpha/r^2$ ($\alpha > 0$).

(a) Determine the effective potential $U_{\text{eff}}(r)$. Sketch it, and discuss qualitatively the trajectories. There are three physically meaningful ranges of E and ℓ, which ones? (2 points)

(b) Write down the equation for $r(t)$ and solve for $t(r)$. With a suitable choice of the origin of the time coordinate the solution reads

$$t = \frac{1}{E}\sqrt{\frac{\mu}{2}}\sqrt{Er^2 - \frac{\ell^2}{2\mu} + \alpha}\,.$$

Under which circumstances does the mass fall from a distance r_0 to the center? How long does this process take? (4 points)