Exercises Electrodynamics	**Sheet 12**

[Tutorial 14.12.]

Exercise 1: We study a metal which has the dielectric function

$$\epsilon(\omega) := 1 - \frac{\Omega^2}{\omega(\omega + i\Gamma)}$$

and the permeability $\mu(\omega) := 1$. The are no external charges, i.e. $\rho_{\text{ext}} = \vec{j}_{\text{ext}} = 0$.

(a) Consider first frequencies $\Gamma \ll \Omega \ll \omega$. Show that the refractive index $n = n_r + i\kappa$ is real in this domain, with $n_r \approx 1$. (Physically, this means that the metal is *transparent* to highly energetic radiation.)

(b) We then move on to $\Gamma \ll \omega \ll \Omega$. Show that $n = n_r + i\kappa$ is purely imaginary in this domain. (Physically, this means that no wave propagation can take place, and the metal reflects waves completely, i.e. functions as a mirror.)

(c) Finally we consider the case $\omega \ll \Gamma \ll \Omega$. A wave moves in the positive z direction. Show that its amplitude gets attenuated as

$$|\vec{E}| \sim e^{-z/d_{\text{skin}}} \ , \quad d_{\text{skin}} := \frac{c}{\sqrt{2\pi\sigma\omega}} \ , \quad \sigma := \frac{\Omega^2}{4\pi\Gamma} \ .$$

This is called the *skin effect*.

Exercise 2: For so-called plasma waves, we can set $\Gamma \to 0$ in the dielectric function from Exercise 1, whereby it takes the form $\epsilon(\omega) := 1 - \Omega^2/\omega^2$, and we also assume $\mu(\omega) = 1$.

(a) Show that the dispersion relation takes the form $\omega^2 = \Omega^2 + c^2k^2$.

(b) Sketch the phase velocity $v_{\text{P}} = \omega/k$ and the group velocity $v_{\text{G}} = \mathrm{d}\omega/\mathrm{d}k$ as functions of ω, and elaborate on the physical meaning of these results.

Exercise 3: Let $\epsilon(\omega) := \epsilon_0 - \Omega^2/(i\omega\Gamma)$ and $\mu(\omega) := \mu_0$. Show that the electric field satisfies the so-called telegraph equation,

$$\left(\nabla^2 - \frac{1}{c_{\text{eff}}^2}\frac{\partial^2}{\partial t^2}\right)\vec{E}(\vec{x},t) = \frac{4\pi\mu_0\sigma}{c^2}\partial_t\vec{E}(\vec{x},t) \ , \quad c_{\text{eff}} := \frac{c}{\sqrt{\epsilon_0\mu_0}} \ , \quad \sigma := \frac{\Omega^2}{4\pi\Gamma} \ .$$

(The origin of this equation comes from the transmission of signals through sea cables.)