Exercises Electrodynamics	**Sheet 11**

[Tutorial 7.12.]

Exercise 1: Consider a charged particle moving with constant velocity v in the x-direction.

(a) Evaluate the Liénard-Wiechert potential from exercise 10.3 for this case.

(b) Compute the Coulomb field of the particle in its rest frame Σ', and boost then to the frame in which it moves. Do you obtain the same result as under point (a)?

Exercise 2: The Liénard-Wiechert potential from exercise 10.3 can also be used for an accelerating particle. This leads to the important phenomenon of *radiation loss*: the accelerating particle loses energy into radiation. The rate at which this happens can be obtained by computing the Poynting vector corresponding to the Liénard–Wiechert potential, however the practical analysis is cumbersome, so we only cite the result: the radiation power per angle is

$$\frac{\mathrm{d}P_0}{\mathrm{d}\Omega} = \frac{q^2|\vec{n} \times [(\vec{n} - \vec{\beta}) \times \dot{\vec{\beta}}]|^2}{4\pi c(1 - \vec{\beta} \cdot \vec{n})^5} \ , \quad \vec{\beta} := \frac{\dot{\vec{r}}}{c}\bigg|_{\tau=\tau_0} \ , \quad \vec{n} := \frac{\vec{x} - \vec{r}}{|\vec{x} - \vec{r}|}\bigg|_{\tau=\tau_0} \ .$$

(a) Consider first the non-relativistic limit, $|\vec{\beta}| \ll 1$. In which direction, compared with the acceleration or deceleration, does most radiation take place?

(b) Consider then acceleration or deceleration in the direction of the velocity, $\dot{\vec{\beta}} \parallel \vec{\beta}$, but with large velocity, $|\vec{\beta}| \gg 1$. In which direction is radiation emitted now? Please sketch the distribution!

Die Prüfung findet am 10.01.2023 um 13:15 - 15:45 Uhr statt, im Hörsaal A6. Keine Hilfsmittel sind erlaubt, aber die folgende Tabelle wird auf dem Prüfungsblatt gegeben:

$\nabla\cdot\vec{E} = 4\pi\rho$	$\nabla\cdot\vec{D} = 4\pi\rho_{\text{ext}}$	$\vec{f}_L = \rho\vec{E} + \frac{1}{c}\vec{j}\times\vec{B}$	$\partial_t\rho + \nabla\cdot\vec{j} = 0$
$\nabla\times\vec{B} - \frac{1}{c}\dot{\vec{E}} = \frac{4\pi}{c}\vec{j}$	$\nabla\times\vec{H} - \frac{1}{c}\dot{\vec{D}} = \frac{4\pi}{c}\vec{j}_{\text{ext}}$	$\frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} = \frac{q}{c}F^{\mu\nu}u_\nu$	$\nabla^2\frac{1}{r} = -4\pi\delta^{(3)}(\vec{r})$
$\nabla\times\vec{E} + \frac{1}{c}\dot{\vec{B}} = \vec{0}$	$\vec{E} = -\nabla\phi - \frac{1}{c}\dot{\vec{A}}$	$\partial_\mu F^{\mu\nu} = \frac{4\pi}{c}J^\mu$	$\Box\frac{\delta(r/c-t)}{r} = 4\pi\delta^{(3)}(\vec{r})\delta(t)$
$\nabla\cdot\vec{B} = 0$	$\vec{B} = \nabla\times\vec{A}$	$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$	$\nabla\times(\nabla\times\vec{B}) = \nabla(\nabla\cdot\vec{B}) - \nabla^2\vec{B}$