Exercises Electrodynamics	**Sheet 10**

[Tutorial 30.11.]

Exercise 1: An infinitely long and thin straight wire is at rest with respect to the inertial system Σ' and carries the charge density $\lambda := Q/\text{length}$. This system moves with respect to the lab frame Σ with the velocity $\vec{v}$, which is parallel to the direction of the wire.

(a) What are $\vec{E}$ and $\vec{B}$ in cylindrical coordinates in Σ'?

(b) Carry out a Lorentz transformation, in order to determine $\vec{E}$ and $\vec{B}$ in Σ.

(c) What are the charge and current density in Σ?

(d) Make use of ρ and $\vec{j}$ in Σ, in order to determine directly $\vec{E}$ and $\vec{B}$ in the lab frame. Do you get the same as under point (b)?

Hint: A Lorentz boost $u^\mu = \Lambda^\mu{}_\nu u'^\nu$ in the x direction with velocity $\vec{v} = v\vec{e}_x$ has the form

$$\Lambda = \begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} , \quad \text{with} \quad \beta := \frac{v}{c} \quad \text{and} \quad \gamma := \frac{1}{\sqrt{1-\beta^2}} .$$

Exercise 2: In the lecture we have obtained the retarded Green's function

$$G_{\text{ret}}(\vec{x}, t) = \frac{\delta(\frac{r}{c} - t)}{r} , \quad r \equiv |\vec{x}| .$$

Show that this can be rewritten in a Lorentz invariant form as $G_{\text{ret}} = 2c\,\theta(t)\delta(x^2)$, $x \equiv (x^0, \vec{x})$.

Exercise 3: In the lecture we have obtained the vector potential corresponding to a general current density as

$$\vec{A}(\vec{x}, t) = \frac{1}{c} \int_V \mathrm{d}^3\vec{x}' \, \frac{\vec{j}(\vec{x}', t - \frac{|\vec{x}-\vec{x}'|}{c})}{|\vec{x}-\vec{x}'|} .$$

Let us now apply this to a single charged particle moving on the trajectory $\vec{r}(t)$, with

$$J^\nu(\vec{x}, t) = q\,\delta^{(3)}(\vec{x} - \vec{r}(t)) \begin{pmatrix} c \\ \dot{\vec{r}}(t) \end{pmatrix} .$$

Show [at least for the spatial components] that this leads to the "Liénard–Wiechert potential"

$$A^\nu(x) = \frac{q u^\nu(\tau_0)}{u(\tau_0) \cdot (x - r(\tau_0))} ,$$

where τ_0 is the Eigenzeit (proper time) at which the signal observed at x was sent.