Exercises Electrodynamics	**Sheet 9**

[Tutorial 23.11.]

Exercise 1: A polarized plane wave has the electric field $\vec{E} = \text{Re}[\vec{\lambda}\, e^{-i(\omega t - \vec{k}\cdot\vec{x})}]$, where $\vec{\lambda}$ is the polarization vector (in the notation of the script, $\vec{\lambda} = i|\vec{k}|\vec{e}$ and $\omega = ck$). The polarization vector is orthogonal to $\vec{k}$, and in general complex; if it is real, the wave is *linearly polarized*.

Consider two plane waves of the same amplitude, phase, and angular frequency. One of them is polarized in the direction of $\vec{e}_y$ and propagates in the negative z-direction; the other is polarized in the direction of $\vec{e} = \cos\alpha\, \vec{e}_y + \sin\alpha\, \vec{e}_z$ and propagates in the positive x-direction. Along the line $\vec{y}(s) = \frac{s}{\sqrt{2}}(\vec{e}_x + \vec{e}_y)$ are placed many detectors close to each other. These measure the time average of the electric field squared, which is proportional to the intensity,

$$I(s) := \langle \vec{E}^2 \rangle := \frac{1}{T}\int_0^T \mathrm{d}t\, \vec{E}^2(\vec{y}(s), t)\ , \quad \omega T \gg 1\ .$$

What is the total electric field $\vec{E}(\vec{x}, t)$? What is $I(s)$? Which distances Δs have the maxima of $I(s)$ along the line? How can one deduce from Δs the wavelength λ? For which α is the interference pattern most pronounced (show a sketch)?

Exercise 2: In the lecture we determined the vector potential of a monochromatic dipole emitter,

$$\vec{A}(\vec{x}, t) = k\vec{P}\,\frac{\sin(-\omega t + kr)}{r}\ , \quad k := \frac{\omega}{c}\ , \quad r := |\vec{x}|\ , \quad r \gg d\ ,$$

where $\vec{P}$ is the electric dipole moment, and d the extension of the emitters, placed at $\vec{x} = \vec{0}$.

(a) Determine the fields $\vec{B}$ and $\vec{E}$.

(b) Show that in the so-called near zone, $kr \ll 1$, the magnetic induction is much smaller than the electric field.

(c) Show that in the so-called far zone, $kr \gg 1$, the typical properties of electromagnetic waves, that is

$$\vec{e}_r \cdot \vec{E} = \vec{e}_r \cdot \vec{B} = \vec{E}\cdot\vec{B} = 0\ ,$$

are satisfied. Here $\vec{e}_r := \vec{r}/r$ is the propagation direction of the outgoing spherical wave.

Exercise 3: A particle is kept on a circular track of radius R with an angular velocity ω, so that the movement is non-relativistic, i.e. $\omega R \ll c$. The charge density can be represented as

$$\rho(\vec{x}, t) = q\,\delta\big(x - R\cos(\omega t + \alpha)\big)\,\delta\big(y - R\sin(\omega t + \alpha)\big)\,\delta(z)\ .$$

(a) Compute the dipole moment of this source, and represent it as $\vec{P} = \text{Re}\,[\vec{P}_0\, e^{-i\omega t}]$.

(b) We now replace one particle by $N \geq 2$ particles, so that their phases are $\alpha_k = 2\pi k/N$, $k = 0, \ldots, N-1$. Show that in this case the dipole moment vanishes. [Remark: an apparent consequence, that there is no radiation, holds only in the non-relativistic limit.]