Exercises Electrodynamics	**Sheet 8**

[Tutorial 16.11.]

Exercise 1: We are studying the time-independent charge distribution

$$\rho(\vec{x}) = \rho_0 \, e^{-\mu|\vec{x}|} \; ,$$

where μ is a constant. The task is to determine the corresponding electric field with the help of Fourier transformations.

(a) What is the Fourier transform $\tilde{\rho}(\vec{k})$ of the charge distribution $\rho(\vec{x})$?

(b) Determine the solution $\tilde{\vec{E}}(\vec{k})$ of the Fourier-transformed Maxwell equation, and compute then $\vec{E}(\vec{x})$ with the inverse transformation.

[Hints: $\int_0^\infty \mathrm{d}r \, r \sin(kr) e^{-\mu r} = \frac{2\mu k}{(k^2+\mu^2)^2}$, $\int_0^\infty \mathrm{d}k \, \frac{\sin(kr)}{k(k^2+\mu^2)^2} = \frac{\pi}{4\mu^4}[2 - e^{-\mu r}(\mu r + 2)]$.]

Exercise 2: An emitter sends plane waves along the positive and negative x-axis,

$$\vec{E}(\vec{x},t) = E_0 \, \vec{e}_y \left[\theta(x) \cos(kx - \omega t) + \theta(-x) \cos(kx + \omega t) \right] \; , \quad \omega = ck \; ,$$

where θ is the Heaviside function. Which current density $\vec{j}(\vec{x},t)$ is needed in order to generate this electric field? [Hint: determine first $\vec{B}(\vec{x},t)$.]

Exercise 3: A plane wave is defined through the potentials

$$\phi = 0 \; , \quad \vec{A} = A(x - ct) \, \vec{e}_z \; .$$

Determine the corresponding $\vec{E}$ and $\vec{B}$, the energy density e_{em}, and the Poynting vector $\vec{S}$.