Exercises Electrodynamics	**Sheet 4**

[Tutorial 19.10.]

Exercise 1: Consider an infinitely long straight wire, which is hollow from the inside (inner radius a), and has a current I running through, with a constant current density between the inner and outer radii $b > a$. Determine the magnetic induction $\vec{B}$ in all domains, by making use of Ampère's law. Sketch the result. [Hint: employ the cylindrical coordinates ρ, φ, z, and use the symmetries of the problem, in order to start with a suitable Ansatz for $\vec{B}$.]

Exercise 2: In the lecture we derived the expression

$$\vec{A}(\vec{x}) = \frac{1}{c} \int_V \mathrm{d}^3\vec{x}' \, \frac{\vec{j}(\vec{x}')}{|\vec{x} - \vec{x}'|}$$

for the vector potential originating from a given current density. Verify that this expression fulfils the Coulomb gauge condition.

Exercise 3: A sphere (radius R, origin at $\vec{x} = \vec{0}$) with a homogeneous surface charge density σ rotates with a constant angular velocity $\vec{\omega} = \omega\, \vec{e}_z$ around the z-axis. The task is to determine the magnetic induction $\vec{B}$ inside and outside of the surface of the sphere.

(a) What are the charge and current density ρ and $\vec{j}$?

(b) A useful Ansatz for the solution turns out to be ($r := |\vec{x}|$)

$$\vec{B} = f(r) z\, \vec{x} - g(r)\, \vec{e}_z \ .$$

To what extent do the sourceless Maxwell equations inside and outside of the surface fix f and g? Two constants stay undetermined; which one vanishes inside, which outside?

(c) Determine the unknown constants. For this you need to inspect the Maxwell equations at the surface, i.e. $r = R$.