Exercises Electrodynamics	**Sheet 3**

[Tutorial 12.10.]

Exercise 1: An infinitely thin round disk of radius R is homogeneously charged with the surface charge density σ, i.e.

$$\rho(\vec{x}) = \sigma\theta(R-r)\delta(z) \ , \quad r := \sqrt{x^2+y^2} \ .$$

(a) Determine the potential $\phi(\vec{x})$ along the z-axis.

(b) Determine the total charge Q, the dipole moment $\vec{P}$, and the quadrupole tensor Q^{ij} of this charge distribution.

(c) Give the multipole expansion of the potential up to the quadrupole term, and compare for $\vec{x} = z\vec{e}_3$ with the exact result from point (a).

Exercise 2: How do the total charge Q, the dipole moment $\vec{P}$, and the quadrupole tensor Q^{ij} transform under

(a) translation of the origin with the vector $\vec{a}$, $\vec{x}' \to \vec{a} + \vec{y}'$?

(b) rotation of the coordinate system with the matrix R?

[Hint: The charge density itself is a scalar quantity, i.e. invariant in the transformation, which we may write in a somewhat sloppy notation as $\rho(\vec{x}') \to \rho(\vec{y}')$.]

Exercise 3: In the script the formula

$$\frac{1}{|\vec{x}-\vec{x}'|} = 4\pi \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{1}{2\ell+1} \frac{r_<^\ell}{r_>^{\ell+1}} Y_{\ell m}^*(\theta',\varphi')\, Y_{\ell m}(\theta,\varphi)$$

has been cited, where $r_< \equiv \min(r,r')$, $r_> \equiv \max(r,r')$, and $Y_{\ell m}$ are spherical harmonics.

(a) Make use of this formula to show that a general solution of the Laplace equation, $\nabla^2\phi = 0$, can be written in spherical coordinates as

$$\phi(r,\theta,\varphi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left(a_{\ell m} r^\ell + \frac{b_{\ell m}}{r^{\ell+1}} \right) Y_{\ell m}(\theta,\varphi) \ .$$

(b) Let us assume that there is an infinitely thin spherical shell at $r = R$ where the potential is $\phi = \phi_0 \sin\theta\cos\varphi$, but the space is empty otherwise. What is ϕ at $r < R$?