Exercises Electrodynamics	**Sheet 2**

[Tutorial 5.10.]

Exercise 1: If charges are placed on an infinitely narrow surface, we define a surface charge density as the charge per area. Use the Gauss law, $\int_{\partial V} \mathrm{d}\vec{f} \cdot \vec{E} = Q_V$, to determine the electric field originating from two charged co-axial cylinders of infinite length and vanishing thickness. The outer cylinder (radius b) has the constant surface charge density $-\sigma$, the inner cylinder (radius $a < b$) the constant surface charge density σ. [Hint: use as integration domain the surface of a cylinder of length L.]

Exercise 2: We want to determine the electric potential of a homogeneously charged infinitely thin straight wire (please choose the z-axis along the wire).

(a) Let us first assume the wire infinitely long, and denote its charge density by $\lambda := Q/\text{length}$. Sketch the vectors of the electric field. Make use of the symmetry of the problem, like in Exercise 1, to deduce the electric field and potential.

(b) Let us now assume that the wire has the length $2a$, and that its center is placed at the origin. Give the charge density in cartesian coordinates.

(c) Determine from the charge density the potential $\phi(\vec{x})$, by making use of a Green's function. Do you recover for $a \to \infty$ the result from point (a)? How can you reproduce the Coulomb potential in the limit $a \to 0$, while keeping $q = 2a\lambda$ fixed?

Exercise 3: Consider the potential

$$\phi = q e^{-2r/a_\mathrm{B}} \left(\frac{1}{r} + \frac{1}{a_\mathrm{B}} \right) ,$$

which describes the hydrogen atom in its ground state (a_B is the Bohr radius and q the elementary charge). Determine the corresponding charge density, and show that the total charge vanishes. [Hint: the handling of the singular point $r = 0$ requires care. The most divergent behaviour can be identified by comparing with the case of a point charge.]