Exercises Electrodynamics	**Sheet 1**

[Tutorial 28.9.]

Exercise 1: The gravitational potential energy of a point mass m_0 at $\vec{x}$ in the presence of another mass m at $\vec{x}'$ is given by

$$U(\vec{x}) = -\frac{Gm_0 m}{|\vec{x} - \vec{x}'|} \,.$$

According to the superposition principle, many masses m_a at $\vec{x}'_a$ produce

$$U(\vec{x}) = -\sum_a \frac{Gm_0 m_a}{|\vec{x} - \vec{x}'_a|} \,.$$

If the point masses are replaced by a continuous mass distribution, we get

$$U(\vec{x}) = -\int \mathrm{d}^3\vec{x}' \, \frac{Gm_0 \rho(\vec{x}')}{|\vec{x} - \vec{x}'|} \,.$$

Let us assume that the mass distribution is spherically symmetric, i.e. that $\rho = \rho(|\vec{x}'|)$.

(a) Show that if $\rho(r) = 0$ for $r > r_0$, then $U(\vec{x})$ at $|\vec{x}| > r_0$ equals the potential originating from a point mass M at origin, whereby M equals the total mass of the body. [Hint: use spherical coordinates for $\vec{x}'$ and place the z-axis in the direction of $\vec{x}$.]

(b) Show that if $\rho(r) = 0$ for $r < r_0$, then $U(\vec{x})$ is constant for $|\vec{x}| < r_0$.

(c) How would you re-interpret the above results in electrodynamics?

Exercise 2: Compute explicitly the surface integral $\int_B \mathrm{d}\vec{f} \cdot \vec{E}$ for $\vec{E}(\vec{x}) := \vec{x}$. The surface B is the half sphere $\vec{x}^2 = 1$, $z > 0$, completed by the bottom $x^2 + y^2 \leq 1$, $z = 0$. Choose cartesian or spherical coordinates, as convenient. Verify the Gauss law.

Exercise 3: Let ϵ_{ijk} be the Levi-Civita tensor and δ_{ij} the Kronecker symbol. We employ the Einstein convention for summation over same indices. Verify the following results:

(a) $\epsilon_{ijk}\epsilon_{lmk} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}, \qquad \epsilon_{ijk}\epsilon_{ljk} = 2\delta_{il}, \qquad \epsilon_{ijk}\epsilon_{ijk} = 6.$

(b) $\vec{a} \cdot \vec{b} = a^i b^j \delta_{ij}, \qquad \vec{a} \cdot (\vec{b} \times \vec{c}) = a^i b^j c^k \epsilon_{ijk}, \qquad \vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\,\vec{b} - (\vec{a} \cdot \vec{b})\,\vec{c}.$

(c) $\nabla \cdot (\vec{a} \times \vec{b}) = \vec{b} \cdot (\nabla \times \vec{a}) - \vec{a} \cdot (\nabla \times \vec{b}).$