Freeze-in dark matter. 2. Sterile neutrinos

As an example of a freeze-in dark matter candidate, we consider the case of „sterile neutrinos", i.e. the heavy eigenstates obtained after adding right-handed neutrino fields to the Standard Model Lagrangian.

Basics: For simplicity, we restrict here to one generation of ν_R. Let

$$\mathcal{L} = \mathcal{L}_{SM} + \bar{\nu}_R i\gamma \nu_R - h_\nu \left(\bar{\nu}_R \tilde{\phi}^\dagger \ell_L + \bar{\ell}_L \tilde{\phi} \nu_R \right) - \frac{M_M}{2} \left(\bar{\nu}_R^c \nu_R + \bar{\nu}_R \nu_R^c \right) .$$

Setting $\tilde{\phi} \to \frac{1}{\sqrt{2}}\begin{pmatrix} v \\ 0 \end{pmatrix}$, $v \simeq 246$ GeV, and denoting $M_D := \frac{h_\nu v}{\sqrt{2}}$, we get the mass terms

$$-\mathcal{L} \supset \bar{\nu}_L M_D \nu_R + \frac{1}{2} \bar{\nu}_R^c M_M \nu_R + \text{H.c.}$$

Inserting $-1 = CC$ and noting that $\nu_R^T C = \bar{\nu}_R^c$, we can write

$$\bar{\nu}_L M_D \nu_R = \frac{1}{2}\left(\bar{\nu}_L M_D \nu_R - \nu_R^T M_D \bar{\nu}_L^T \right) = \frac{1}{2}\left(\bar{\nu}_L M_D \nu_R + \bar{\nu}_R^c M_D \nu_L^c \right),$$

whereby the mass terms become

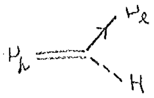
$$-\mathcal{L} \supset \frac{1}{2}\left(\bar{\nu}_L \;\; \bar{\nu}_R^c \right) \underbrace{\begin{pmatrix} 0 & M_D \\ M_D & M_M \end{pmatrix}}_{\mathcal{M}_\nu} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + \text{H.c.}$$

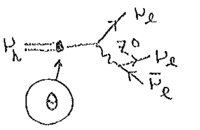
The symmetric matrix $\mathcal{M}_\nu$ can be represented as $\mathcal{M}_\nu = U \begin{pmatrix} m_\nu & \\ & M \end{pmatrix} U^T$, where $m_\nu, M > 0$. This can most easily be implemented by treating the Hermitean matrix $\mathcal{M}_\nu \mathcal{M}_\nu^\dagger = U \begin{pmatrix} m_\nu^2 & \\ & M^2 \end{pmatrix} U^\dagger$ with standard diagonalization. If $M_D \ll M_M$, we get

$$m_\nu \approx \frac{M_D^2}{M_M}, \quad M \approx M_M, \quad U \simeq \begin{pmatrix} 1 & \theta \\ -\theta & 1 \end{pmatrix}, \quad \theta \approx \frac{M_D}{M_M} .$$

So we have the constraint $\boxed{\frac{h_\nu^2 v^2}{M} \lesssim \text{eV}} \Leftrightarrow h_\nu^2 \lesssim 10^{-14} \frac{M}{\text{GeV}}$ (a)

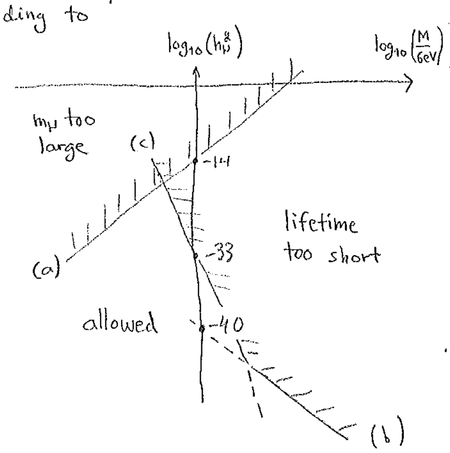
Stability: The heavy eigenstates ($=: \nu_h$) are <u>unstable</u>, decaying into light ones ($=: \nu_\ell$).

$M > m_H$:  $\Gamma \sim \frac{h_\nu^2 M}{8\pi}$

$M \ll m_H$:  $\Gamma \sim \frac{G_F^2 M^5 \theta^2}{96\pi^3} \sim \frac{G_F^2 M^3 h_\nu^2 v^2}{96\pi^3}$.

Inverse of the lifetime of the universe : $\frac{1}{10^{10}\text{y}} \sim \frac{1}{10^{17}\text{s}} \sim 10^{-32}\,\text{eV} =: \tau^{-1}$.
We have to require $\Gamma < \tau^{-1}$, leading to

$$\begin{cases} h_\nu^2 \frac{M}{\text{GeV}} < 10^{-40}, & M > m_H \;\text{(b)} \\ h_\nu^2 \left(\frac{M}{\text{GeV}}\right)^3 < 10^{-33}, & M \ll m_H \;\text{(c)} \end{cases}$$



If we want to lie on the curve (a), accounting for neutrino masses, we need

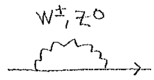
$$\frac{M}{\text{GeV}} \lesssim 10^{-5}, \text{ or } M \ll \text{MeV}.$$

How to estimate Γ?

As the right-handed states are singlets, they should be produced through an oscillation from left-handed neutrinos (like decay on p.25 but in opposite direction):

$\underbrace{\qquad}_{\nu_L \text{ produced through Fermi vertex}}$ $\underbrace{\qquad}_{\text{virtual } \nu_L \text{ propagates}}$ $\underbrace{\qquad}_{\text{on-shell } \nu_R \text{ as final state}}$

The process is complicated by the fact that ν_L propagates in a medium, whereby its propagator gets modified.*



* This is similar in spirit to the so-called MSW-effect (Mikheyev - Smirnov - Wolfenstein) that is discussed in connection with solar and atmospheric neutrino oscillations.

Squaring the amplitude, we can estimate (assuming $\pi T \gg M$)

$$\Gamma \sim \underbrace{10 G_F^2 T^4 k}_{\text{from } |\times|^2} \; \underbrace{\frac{M^4}{(M^2 + 100 G_F^2 T^4 k^2)^2}}_{\text{from medium modification of the propagator}} \cdot \underbrace{\theta^2}_{\text{from mixing}}$$

Factors of k require an explicit computation. Setting $k \sim T$ and dividing by $H \sim T^2/m_{Pl}$, we get

$$\hat{\Gamma} \sim \frac{10 G_F^2 T^{-3} m_{Pl} M^2 h_\nu^2 v^2}{(M^2 + 100 G_F^2 T^6)^2} .$$

Relic abundance

$\hat{\Gamma}$ decreases at small T as $\sim T^3$ and at large T as $\sim T^{-9}$. The peak is where $M \sim 10 G_F T^3$, i.e.

$$T \sim \left(\frac{M}{10 G_F}\right)^{\frac{1}{3}} \sim \left(\frac{M}{\text{MeV}}\right)^{\frac{1}{3}} \left(\frac{10^{-3}\,\text{GeV}^3}{10 \cdot 10^{-5}}\right)^{\frac{1}{3}} \sim 2\,\text{GeV}\left(\frac{M}{1\text{keV}}\right)^{\frac{1}{3}}.$$

The peak value is

$$\hat{\Gamma}_{max} \sim \frac{G_F m_{Pl} M^3 h_\nu^2 v^2}{M^4} \sim \frac{G_F m_{Pl} h_\nu^2 v^2}{M} \lesssim G_F m_{Pl} m_\nu$$

$$\sim 10^{-5}\,\text{GeV}^{-2}\, 10^{19}\,\text{GeV}\, 10^{-9}\,\text{GeV} \sim 10^5 \;!$$

So we see that equilibration is possible if h_ν^2 is maximally large, otherwise it's not guaranteed.

The final Y needed (p.16 with TeV $= 10^6$ MeV) is

$$Y \cdot M \sim 4 \times 10^{-7}\,\text{MeV} = 4 \times 10^{-4}\,\text{keV} = 0.4\,\text{eV}$$

If there is equilibration, we expect $Y \sim \frac{1}{g_*} \sim 10^{-2} \ldots 10^{-1}$. We see that this would be problematic unless $M \lesssim$ eV.

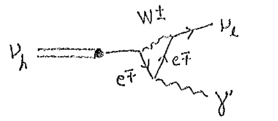
Summary: If we account for m_ν, only very small masses are allowed (but these are excluded from other considerations, see p.27). If we decrease h_ν^2 and do not account for m_ν, there is more freedom.**

** This general scenario was proposed by S. Dodelson and L.M. Widrow, „Sterile neutrinos as dark matter", hep-ph/9303287, building on earlier studies like K. Enqvist, K. Kainulainen, M. Thomson, Nucl. Phys. B 373 (1992) 498

<u>Constraints:</u> (i) Staying within the allowed window from p. 25, $\hat{\Gamma}_{max}$ does in general not reach unity*, and a numerical integration of the rate equation is required (cf. p. 21). This yields a line in the (M, h_ν^2)-plane which gives the correct dark matter abundance.

* At least not for a long time.

(ii) Apart from the decays discussed on p. 25, there are also decays which would produce photons:



$$\Gamma_\gamma \sim \frac{G_F^2 M^5 \theta^2}{256\pi^4}\,\alpha_{em} .$$

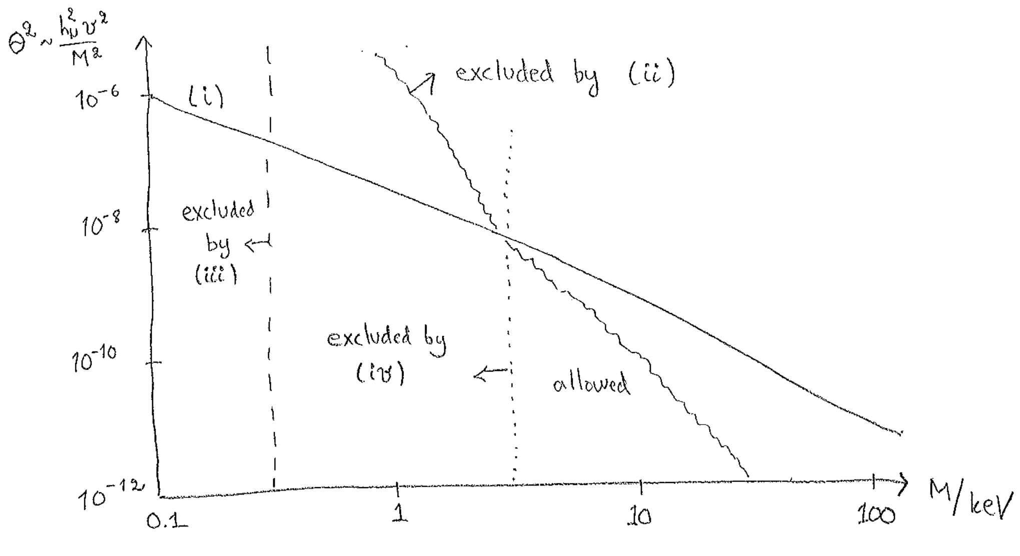
Even if $\Gamma_\gamma \ll \Gamma$, some decays would happen in regions containing lots of dark matter (e.g. galactic centers). As no signal has been observed, there is an <u>observational upper bound</u> on $M^5\theta^2 \sim M^3 h_\nu^2$ from here.

(iii) Sterile neutrinos are fermions. Pauli principle prohibits us from packing them together too densely. Multiplying the maximal density by the mass, and comparing with maximal known dark matter densities, sets a <u>lower bound</u> on the mass of a fermionic dark matter candidate, known as the Tremaine-Gunn bound**:

$$M \gtrsim 0.3\,\text{keV}.$$

** S. Tremaine and J.E. Gunn, „Dynamical role of light neutral leptons in cosmology", Phys. Rev. Lett. 42 (1979) 407

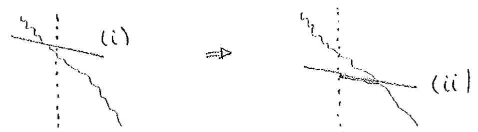
(iv) Stronger but more model-dependent <u>lower bounds</u> are obtained by studying large-scale structure formation at early times, particularly through the so-called „Lyman-α forest" (cf. p. 11): $M \gtrsim 3$ keV.



<u>Summary:</u> There is an allowed range with $M \gtrsim 3$ keV and $h_\nu^2 \lesssim 10^{-8}\frac{M^2}{v^2} \sim 10^{-8}\left(\frac{10^{-6}\,\text{GeV}}{10^2\,\text{GeV}}\right)^2 \sim 10^{-24}$, but according to the theoretical computation, <u>too little dark matter</u> is produced for such h_ν^2.

Modern challenges:

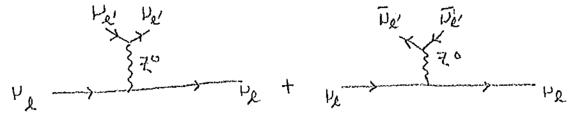
Like with freeze-out dark matter on p.18, we would like to increase Γ. On p.18 this was needed for dark matter to reduce its abundance, thereby avoiding overclosure. Now it is needed for producing enough dark matter even with a smaller θ^2:

On p.27: 

$\uparrow h_\mu^2$ or θ^2

no solution exists — viable solutions exist

How could this be done? An interesting idea*: if the cosmological background is different, including a substantial lepton asymmetry ($\mu_L \neq 0$), then there is an additional contribution to the active neutrino self-energy:

e.g.: 

There is a contribution which does not cancel if $\mu_L \neq 0$,** resulting in

$$\Gamma \sim 10 G_F^2 T^4 k \frac{M^4}{(M^2 - G_F T^2 \mu_L k + 100 G_F^2 T^4 k^2)^2} \theta^2 .$$

We now see that there can be a cancellation in the denominator; in principle there could even be a zero, meaning that Γ becomes very large. We say that the virtual ν_ℓ goes „on-shell", thanks to the medium effects, or becomes „resonant" with the heavy ν_h.

It is an active topic of research whether sufficiently large lepton asymmetries can indeed be produced by other mechanisms taking place in the early universe (p.41–44).

* X.-D. Shi and G.M. Fuller, „A new dark matter candidate: non-thermal sterile neutrinos", astro-ph/9810076

** D. Nötzold and G. Raffelt, „Neutrino dispersion at finite temperature and density", Nucl. Phys. B 307 (1988) 924