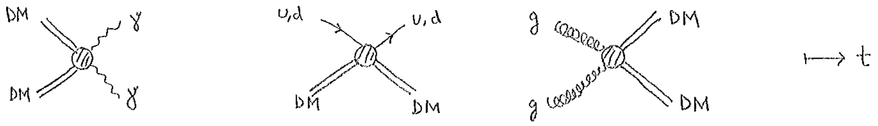
Freeze-out dark matter. 2. Modern challenges

Empirical constraints: Dark matter plays a role not only in cosmology but should also be around today, and can be searched for. Typical processes:



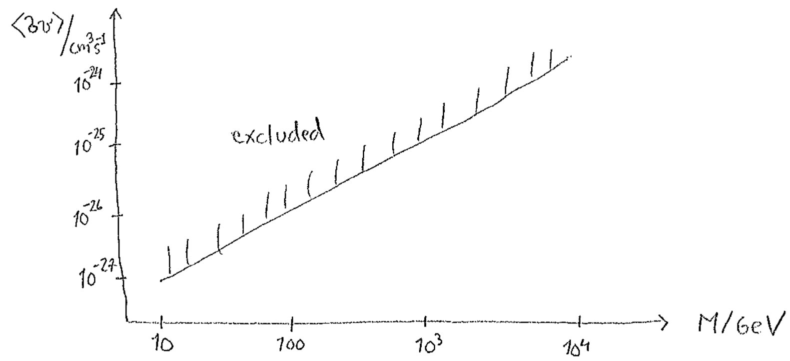
(i) „indirect detection" from galactic center

(ii) „direct detection" in underground lab

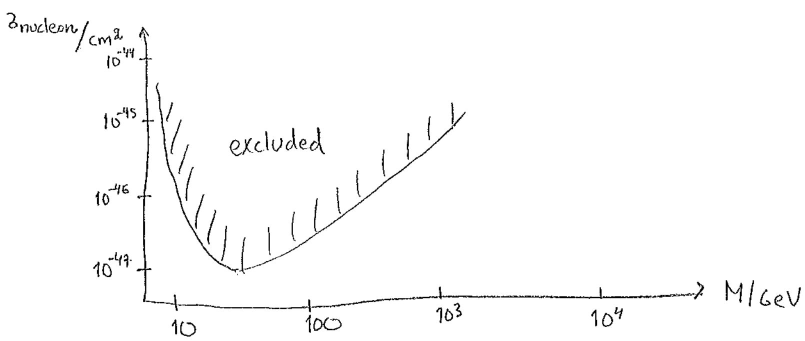
(iii) „missing energy" in collider search

Here some sketches of recent results:

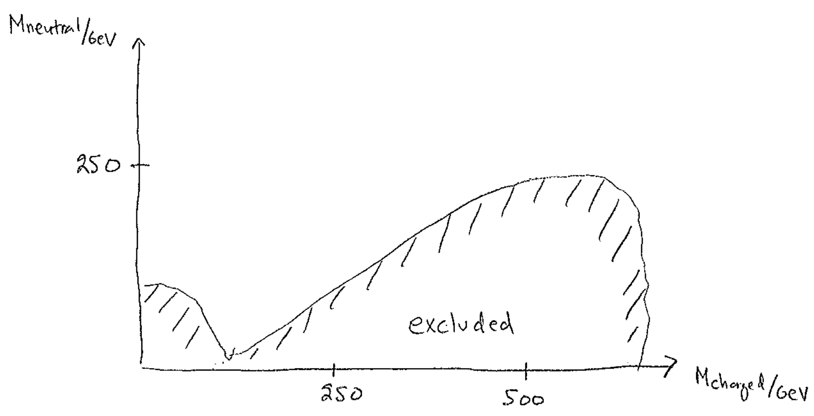
(i) Fermi Large Area Telescope (LAT), 1704.03910 :



(ii) XENON1T experiment, 1805.12562 :



(iii) ATLAS experiment, 1708.07875 :

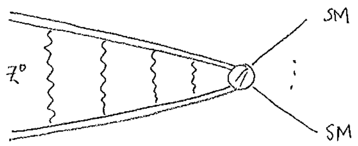


Summary: small masses / large cross sections are disfavoured.

Main challenge: In order to satisfy experimental constraints (p.17), we need to push up the mass scale M and decrease various dark matter cross sections. These requirements are compatible with each other, given that dimensionally $\sigma \sim \alpha^2/M^2$, see p.15. However, as a result dark matter freezes out early, and carries too much energy density, leading to the problem of overclosure (p.16).

Possible solution: Overclosure could be avoided if we could increase $\langle \sigma v \rangle$.

Sommerfeld effect: As an example of an interesting idea, let the dark matter particles interact before their pair annihilation:



Assume an attractive Yukawa potential, $V(r) = -\frac{\alpha}{r} e^{-m_Z r}$. If the dark matter mass scale is M, the Bohr radius is $a = \frac{2}{M\alpha}$. If $m_Z a \ll 1$, i.e. $M \gg \frac{2 m_Z}{\alpha}$, the Yukawa screening is unimportant; for instance, bound states exist if $M > \frac{1.6 m_Z}{\alpha}$.

Now, as the dark matter particles are heavy $(T \sim \frac{M}{25})$, they are non-relativistic $\left(M v^2 \sim T \Rightarrow v \sim \sqrt{\frac{T}{M}} \ll 1 \right)$, and we can describe the annihilation with quantum mechanics.

Annihilation requires that particles meet, i.e. $\sigma \propto |\Psi(\vec{0})|^2$, where Ψ is a non-relativistic wave function. We need to know how $|\Psi(\vec{0})|^2$ changes because of $V(r)$. Let v be the relative velocity, $\vec{v} := \frac{\vec{p} - \vec{q}}{2M}$, and $M_r := \frac{M}{2}$ the reduced mass of the 2-body problem. Parametrizing the relative energy as $E = M v^2$, the Schrödinger equation reads

$$\left[-\frac{\nabla^2}{M} + V(r) \right] \Psi(\vec{r}) = M v^2 \, \Psi(\vec{r}) .$$

The solution was found long ago* for $V(r) \approx -\frac{\alpha}{r}$:

$$S(v) := \frac{|\Psi^2(\vec{0})|}{|\Psi^2(\vec{0})|_{\alpha = 0}} = \frac{\pi\alpha / v}{1 - e^{-\pi\alpha/v}} .$$

We see that $S(v) \approx 1$ if $v \gg \pi\alpha$ but $S(v) \approx \frac{\pi\alpha}{v} \gg 1$ if $v \ll \pi\alpha$.

Then the cross section from scattering states would be

$$\sigma v = (\sigma v)_{\text{tree}} \times S(v) ,$$

and the thermal average reads

$$\langle \sigma v \rangle \approx \frac{\int_{\vec{v}} \sigma v \, e^{-Mv^2/T}}{\int_{\vec{v}} e^{-Mv^2/T}} .$$

If $\sqrt{\frac{T}{M}} \ll \pi\alpha$, this could be larger** than just $\sim \frac{\alpha^2}{M^2}$.

* G. Gamow, Z. Physik 51 (1928) 204; A. Sommerfeld, Ann. Phys. (Leipzig) 403 (1931) 257; A.D. Sakharov, Zh. Eksp. Theor. Fiz 18 (1948) 631; Landau-Lifshitz, Quantum Mechanics, Non-relativistic Theory, § 136

** Actually, the annihilating pairs could also form bound states, which leads to an even larger enhancement.

Remaining challenge: If we increase $\langle \sigma v \rangle$ in cosmology, would we not also increase it in the galactic center (point (i) on p. 17), where the average velocity is even smaller?

Possible solution: Instead of a single dark matter particle, consider a whole „dark sector". Then it can be arranged that some fast annihilation channel is only open in the early universe.

Example of a model: Complement SM with a neutral fermion χ of mass M, and a strongly interacting scalar η of mass $M+\Delta M$:

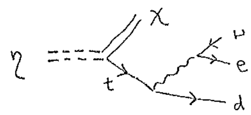
[e.g. M. Garny et al, 1802.00814]

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}\bar{\chi}(i\not{\partial} - M)\chi + (D_\mu\eta)^\dagger(D^\mu\eta) - (M+\Delta M)^2\eta^\dagger\eta$$

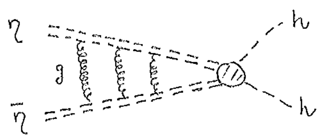
$$- \underbrace{\lambda\, \eta^\dagger\eta}_{\text{„Higgs portal" coupling}} \underbrace{H^\dagger H}_{\text{SM Higgs}} - \underbrace{y\, \eta^\dagger \bar{\chi} t_R}_{\text{„Yukawa portal" coupling}} - y^*\, \underbrace{\bar{t}_R}_{\text{SM top quark}} \chi\eta \,.$$

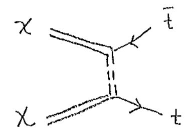
* η is often called a „mediator", since it mediates interaction between the dark sector and SM through the portal couplings

There is a discrete symmetry which prevents χ from decaying, however η can decay:



In the early universe, when $T \gg \Delta M$, several channels are open:





fast because of Sommerfeld effect & bound states

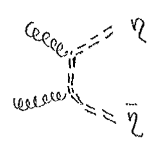
Slow if $|y|^2$ is small (and also because of additional „p-wave" suppression by v^2)

In the late universe, when $T \ll \Delta M$, η has decayed, so that only the slow channel is open, and $\langle \sigma v \rangle$ is small.

Of course we should also take care to respect the other constraints on p. 17:

(ii) „direct detection": since nuclei are made predominantly of u and d quarks, and χ can be tuned to couple mostly to t, σ_{nucleon} can be made small.

(iii) „collider search": at the LHC it would be easy to produce $\eta\bar{\eta}$:



None have been seen $\Rightarrow$ $M + \Delta M \gtrsim 2$ TeV.

Summary: it is quite hard to build a viable model!

Dangerous relics: The problem we have seen with dark matter, namely that adding stable heavy particles to the Standard Model may overclose the Universe, is quite generic and excludes many naive constructions. Let us illustrate this with another consideration, historically known as the „monopole problem".

Premise: suppose there is a stable particle of mass M, produced at a temperature $T_* \sim M$. At this time the age of the universe is $t_* \sim m_{Pl}/T_*^2$ (p.3). Let us assume that there is just one such particle per causally connected region, i.e. „horizon", with all the other ones having pair-annihilated. The corresponding distance scale is denoted by $r_* \sim t_* \sim m_{Pl}/T_*^2$.

Initial density: $n_* \sim \frac{1}{r_*^3} \sim \left(\frac{T_*^2}{m_{Pl}}\right)^3$.

Afterwards distance scales redshift with factor $a(t)$.

Density today $(t = t_0)$: $n_0 \sim n_* \frac{a^3(t_*)}{a^3(t_0)}$.

Yield today:

$$Y = \frac{n_0}{s_0} \sim \left(\frac{T_*^2}{m_{Pl}}\right)^3 \frac{a^3(t_*)}{a^3(t_0)\, s(T_0)}$$

$$= \left(\frac{T_*^2}{m_{Pl}}\right)^3 \underbrace{\frac{a^3(t_*)\, s(T_*)}{a^3(t_0)\, s(T_0)}}_{1 \text{ by entropy conservation}} \cdot \frac{1}{s(T_*)} .$$

We recall from p. 2 that $s(T_*) = g_* \frac{4\pi^2 T_*^3}{90}$.

So we can estimate today's energy density normalized to entropy as

$$\frac{M n_0}{s_0} \sim M \left(\frac{T_*}{m_{Pl}}\right)^3 \frac{T_*^3}{s(T_*)} \sim M \left(\frac{M}{m_{Pl}}\right)^3 .$$

The correct value is (p. 16) $M Y_0 \approx 4.4 \times 10^{-13}$ TeV.

So the model is allowed if

$$\frac{M^4}{m_{Pl}^3} < 4.4 \times 10^{-13}\,\text{TeV}$$

$$\Leftrightarrow \quad M < \Big\{ 4.4 \times 10^{-13} \underbrace{\text{TeV}}_{10^3\,\text{GeV}}\ \underbrace{m_{Pl}^3}_{(10^{19})^3\,\text{GeV}^3} \Big\}^{\frac{1}{4}} \sim 10^{12}\,\text{GeV}.$$

Many GUTs produce monopoles and require $M \gtrsim 10^{15}\,\text{GeV} \Rightarrow \lightning$.

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