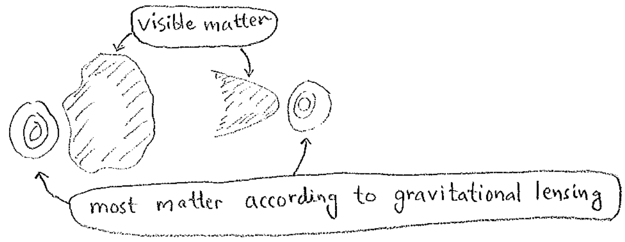
Freeze-out dark matter. 1. Basic picture

We move on to consider yet another decoupling, related to dark matter. Unlike the previous decouplings, this is related to yet unknown physics, so we consider several different types of models.

Empirical facts

* the need for an invisible additional source of gravitational pull originates from the observation of many different systems/phenomena:
  - (i) rotation velocities of stars and gas clouds around galaxies (distance scale $\sim 10 \div 100$ kpc, pc $\approx$ 3ly ; happening today)
  - (ii) rotation of galaxies around cluster of galaxies (distance scale ~ 10 Mpc)
  - (iii) collisions of clusters of galaxies („bullet cluster")*:



  - (iv) CMB: correct temperature anisotropies only produced with dark matter (distance scale ~ Gpc ; happened 13×10^9 y ago)
  - (v) LSS: first galaxies form soon after recombination, so some gravitational collapse should start when the universe was in a plasma phase — this requires dark matter.

* given that we do not see dark matter, it should be neutral (otherwise, there are changing currents and radiation)

* given that dark matter was around long ago and is still today, it should be stable or very long-lived

* to get correct LSS, dark matter should not be too light, as light particles move fast and are difficult to keep together

* the total energy fraction of dark matter today is ~25%, whereas known forms of matter („baryonic matter") carry ~5%

* in absolute terms: $e_{DM}^{(global)} \sim 0.1 \frac{\text{GeV}}{\text{m}^3}$, $e_{DM}^{(local)} \sim 0.1 \frac{\text{GeV}}{\text{cm}^3}$.

There is no stable neutral massive particle in the Standard Model! We are perhaps missing a new Weakly Interacting Massive Particle (WIMP).

* D. Clowe et al, A direct empirical proof of the existence of DM, astro-ph/0608407

Theoretical description

Let us start with the general form of a rate equation (p.6):

$$\frac{df(x,k_T)}{dx} \approx -\hat{\Gamma}(k_T)\left[f(x,k_T) - f_{eq}(\omega_T)\right] \;;\quad k_T = k(T_0)\frac{a(T_0)}{a(T)}\,,$$

$$\hat{\Gamma} = \frac{\Gamma}{\mathcal{J}}\,,\quad f_{eq} \in \{n_F, n_B\}$$

In order to simplify the situation, we assume the particles to be in <u>kinetic equilibrium</u> (i.e. momenta are thermally distributed):

$$f(x,k_T) \approx f_{eq}(\omega_T)\,\frac{n(x)}{n_{eq}(x)}\,,\qquad n_{eq}(x) := \int_{\vec{k}_T} f_{eq}(\omega_T)\,.$$

Let us insert this into the equation, and integrate over $\vec{k}_T$:

$$\int_{\vec{k}_T}\frac{d}{dx}\left\{f_{eq}(\omega_T)\,\frac{n(x)}{n_{eq}(x)}\right\} = -\underbrace{\int_{\vec{k}_T}\hat{\Gamma}(k_T)\,\frac{f_{eq}(\omega_T)}{n_{eq}(x)}}\left[n(x) - n_{eq}(x)\right]$$

denote by $\langle\hat{\Gamma}\rangle := \dfrac{\int_{\vec{k}_T}\hat{\Gamma}(k_T)\,f_{eq}(\omega_T)}{\int_{\vec{k}_T} f_{eq}(\omega_T)}$

Here we have to watch out, because $\vec{k}_T$ depends on $x = \ln\left(\frac{T_{max}}{T}\right)$.
Trick $(\vec{k}_0 := \vec{k}(T_0))$:

$$\int_{\vec{k}_T}\frac{d}{dx}\left\{f_{eq}(\omega_T)\,\frac{n(x)}{n_{eq}(x)}\right\} = \int_{\vec{k}_0}\frac{a^3(T_0)}{a^3(T)}\,\frac{d}{dx}\left\{f_{eq}\left(\sqrt{M^2 + k_0^2\frac{a^2(T_0)}{a^2(T)}}\right)\frac{n(x)}{n_{eq}(x)}\right\}$$

$$= \frac{\cancel{a^3(T_0)}}{a^3(T)}\,\frac{d}{dx}\left\{\frac{a^3(T)}{\cancel{a^3(T_0)}}\underbrace{\int_{\vec{k}_T} f_{eq}(\omega_T)}_{n_{eq}(x)}\frac{n(x)}{n_{eq}(x)}\right\}$$

$\boxed{s(T)a^3(T) = \text{const.}}$

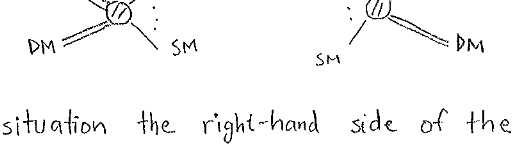
$$= \frac{s(T)}{\cancel{\text{const.}}}\,\frac{d}{dx}\left\{\frac{\cancel{\text{const.}}}{s(T)}\,n(x)\right\}.$$

Finally we define a <u>yield parameter</u> as $Y(x) := \dfrac{n(x)}{s(T)}$, and divide on both sides by $s(T)$. This produces a <u>momentum-averaged evolution equation</u>:

$$Y'(x) \approx -\langle\hat{\Gamma}\rangle\left[Y(x) - Y_{eq}(x)\right]\,,\qquad Y_{eq} := \frac{n_{eq}(x)}{s(T)}\,.$$

R-parity

In order to guarantee the stability of the dark matter particles, they are sometimes postulated to be odd in a discrete symmetry, called R-parity, whereas Standard Model particles are even. Then the lightest R-odd particle <u>cannot decay</u>. Its abundance can change only through pair annihilation or creation:



In this situation the right-hand side of the rate equation should be <u>quadratic in Y</u>.

Lee-Weinberg equation

In order to impose quadratic dependence, we recall from p. 5 that we had omitted terms of higher orders in deviations from thermal equilibrium. Such terms can now be made use of:

$$Y^2 - Y_{eq}^2 = (Y - Y_{eq})(Y + Y_{eq}) = (Y - Y_{eq})(Y - Y_{eq} + 2Y_{eq})$$
$$= 2Y_{eq}(Y - Y_{eq}) + (Y - Y_{eq})^2$$
$$\Leftrightarrow \quad Y - Y_{eq} = \frac{Y^2 - Y_{eq}^2}{2Y_{eq}} + O(Y - Y_{eq})^2 .$$

So, close to equilibrium,

$$Y'(x) \approx -\frac{\langle \hat{\Gamma} \rangle}{2Y_{eq}} \left[Y^2(x) - Y_{eq}^2(x) \right] .$$

Suppose that we furthermore „undo" the variable changes made:

$$Y'(x) = \underbrace{\frac{dt}{dx}}_{1/J} \frac{d}{dt} \left\{ \frac{n}{s} \cdot \frac{a^3(t)}{a^3(t)} \right\} = \frac{1}{J} \cdot \frac{\dot{n} a^3 + 3 n a^2 \dot{a}}{\underset{\text{Const.}}{s}\, a^3} = \frac{1}{Js}(\dot{n} + 3Hn)$$

multiply by Js

$$\Rightarrow \quad \dot{n} + 3Hn \approx -\frac{\langle \Gamma \rangle}{2n_{eq}} (n^2 - n_{eq}^2) .$$

The dimension of $\frac{\langle \Gamma \rangle}{2n_{eq}}$ is $\frac{m^3}{s} = m^2 \cdot \frac{m}{s} = [\text{area}][\text{velocity}]$, and this combination is usually denoted by $\langle \sigma v \rangle$. Thus we get the Lee-Weinberg equation*:

$$\dot{n} + 3Hn \approx -\langle \sigma v \rangle (n^2 - n_{eq}^2) , \qquad \langle \sigma v \rangle := \frac{\langle \Gamma \rangle}{2n_{eq}} .$$

*B.W. Lee and S. Weinberg, Cosmological lower bound on heavy neutrino masses, Phys. Rev. Lett. 39(1977)165; J. Bernstein, L.S. Brown and G. Feinberg, PRD 32(1985)3261.

Approximate solution

Once again we consider decoupling, taking place when $\langle \hat{\Gamma} \rangle \sim 1$. From above we have

$$\langle \Gamma \rangle = 2n_{eq} \langle \sigma v \rangle \overset{\langle \hat{\Gamma} \rangle \sim 1}{\sim} H .$$

Let us parametrize* $\langle \sigma v \rangle =: \frac{\alpha^2}{M^2}$, where M is the dark matter mass scale and $\alpha \sim 0.01$ could be the weak fine-structure constant. The Hubble rate $H \sim \frac{T^2}{m_{Pl}}$ (p. 3) is small, so we need small n_{eq}. This is the case in the non-relativistic regime (p. 2):

$$n_{eq} \sim \left(\frac{MT}{2\pi} \right)^{3/2} e^{-M/T} .$$

*This is in analogy with Thomson scattering, cf. p. 9, but holds also for non-relativistic pair annihilation

So we get

$$\left(\frac{MT}{2\pi} \right)^{3/2} e^{-\frac{M}{T}} \frac{\alpha^2}{M^2} \sim \frac{T^2}{m_{Pl}}$$
$$\Leftrightarrow \quad \frac{M}{T} \sim \ln \left\{ \frac{\alpha^2}{(2\pi)^{3/2}} \cdot \frac{m_{Pl}}{M^{1/2}\, T^{1/2}} \right\}$$

→

Solve this iteratively (cf. p.10), setting $\alpha \sim 0.01$, $M \sim T \sim$ TeV, $m_{Pl} \sim 10^{19}$ GeV:

$$\frac{M}{T} \sim \ln \left\{ \frac{10^{-4}}{(2\pi)^{3/2}} \; \frac{10^{19}}{10^{3}} \right\} \sim \ln\{10^{11}\} \sim 25.$$

Again decoupling takes place surprisingly late ($T \ll M$).

The corresponding yield parameter:

$$Y \sim \left. \frac{n_{eq}}{s} \right|_{\frac{M}{T} \sim 25} \overset{p.2}{\sim} \frac{\left(\frac{MT}{2\pi}\right)^{3/2} e^{-M/T}}{g_* \frac{4\pi^2 T^3}{90}} \sim \frac{1}{g_*} \left(\frac{M}{T}\right)^{3/2} e^{-\frac{M}{T}} \sim 10^{-10}.$$

The correct observed value corresponds to ($Y_0 = Y(T_0)$, $T_0 \sim 10^{-3}$ eV)

$$Y_0 \cdot M \approx 4.4 \times 10^{-13} \text{ TeV}.$$

So if $M \sim$ TeV, we produced <u>too much</u> dark matter!

<u>Sketch of numerical solution</u>