Other decouplings: CMB, 21cm line, BBN

The basic steps of any cosmological computation should have become clear with neutrinos:

(i) write down the „would-be" equilibrium phase space distribution (or „density matrix") ;

(ii) compute the interaction rate Γ at which the system approaches the equilibrium form ;

(iii) from the condition $\Gamma \sim H$, determine when the system decouples from equilibrium.

We now briefly sketch the application of these steps to three problems whose detailed analysis is rather complicated.

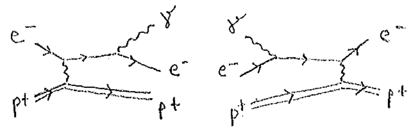
Photon decoupling (also known as last scattering, or recombination)

(i) the equilibrium distribution is $f(k) = n_B(k) \equiv \dfrac{1}{e^{k/T}-1}$.

(ii) photons scatter on charged particles, most importantly via Thomson scattering:

$\sigma \sim \dfrac{8\pi \alpha_{em}^2}{3\, m_e^2}$

They can be produced or absorbed :



Scatterings become rare when e^- and p^+ combine to neutral hydrogen.
The rate of Thomson scatterings depends on the density of electrons:

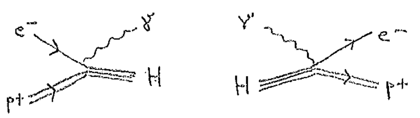
$$\underset{(1/s)}{\Gamma} \sim \underset{(1/m^3)(m^2)(m/s)}{n_e \sigma v} \quad , \quad v \simeq 1 \ .$$

The main challenge is therefore to estimate $n_e(T)$.

* for a precise treatment, electron density should also be treated as a non-equilibrium variable, and solved for from a rate equation

Let us assume that electrons are in equilibrium*. Their number density can change if electrons and protons combine into hydrogen, or vice versa:

$$e^- + p^+ \leftrightarrow H + \gamma$$



These processes release/require substantial energy ($\Delta E = 13.6\,\text{eV}$), but there are many photons around, so energetic ones can be found in the tail of the Bose distribution, even if $T \ll \Delta E$.

Ingredients for determining n_e:

* the baryon density n_B is known: $n_B = \underbrace{\dfrac{n_B}{n_\gamma}}_{6\times 10^{-10},\ \text{cf. p.12}} \cdot \underbrace{n_\gamma}_{2\int_k \frac{1}{e^{k/T}-1} = \frac{2\zeta(3)T^3}{\pi^2}}$

* protons and/or hydrogen carry most of n_B (cf. p. 12):
$$n_p + n_H \approx 0.75\, n_B \ . \qquad (**)$$

* charge neutrality implies $n_e = n_p$.

* as processes like $e^+e^- \leftrightarrow \gamma\gamma$ are not in equilibrium at $T \ll m_e$, chemical potentials need to be introduced for e, p, H.

The key ingredient is the relation of chemical potentials:

$$e^- + p^+ \leftrightarrow H + \gamma \quad \& \quad \mu_\gamma = 0 \quad \Rightarrow \quad \mu_e + \mu_p = \mu_H . \qquad (***)$$

Using (cf. p. 2)

$$n_e = g_e \left(\frac{m_e T}{2\pi}\right)^{3/2} e^{\frac{\mu_e - m_e}{T}} ,$$

and similarly for n_p, n_H, we can proceed as follows:

$$(***) \Rightarrow \quad e^{\frac{\mu_e}{T}} e^{\frac{\mu_p}{T}} = e^{\frac{\mu_H}{T}}$$

$$\Leftrightarrow \quad \frac{n_e}{g_e}\left(\frac{2\pi}{m_e T}\right)^{3/2} e^{\frac{m_e}{T}} \; \frac{n_p}{g_p}\cancel{\left(\frac{2\pi}{m_p T}\right)^{3/2}} e^{\frac{m_p}{T}} = \frac{n_H}{g_H}\cancel{\left(\frac{2\pi}{m_H T}\right)^{3/2}} e^{\frac{m_H}{T}}$$

$$\Rightarrow \quad n_H \approx n_e^2 \frac{g_H}{g_e g_p}\left(\frac{2\pi}{m_e T}\right)^{3/2} e^{\frac{\Delta E}{T}} , \qquad \Delta E = m_e + m_p - m_H \approx 13.6\,\mathrm{eV}.$$

Inserting into eq. (**) we get the <u>Saha equation</u> for n_e:

$$\underbrace{n_e}_{n_p} + n_e^2 \frac{g_H}{g_e g_p}\left(\frac{2\pi}{m_e T}\right)^{3/2} e^{\frac{\Delta E}{T}} \approx 0.75\, n_B .$$

At late times, when $T \ll \Delta E$, the latter term dominates, and

$$n_e^2 \approx 0.75\, n_B \frac{g_e g_p}{g_H}\left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-\frac{\Delta E}{T}}$$

(iii) Let us insert this into $\Gamma \sim n_e \frac{\alpha_{em}^2}{m_e^2} \sim H \sim \frac{T^2}{m_{Pl}}$. $\quad (\hat{\Gamma} \sim 1)$

For simplicity we omit all numerical factors.

$$\Rightarrow \quad n_B^{1/2} (m_e T)^{3/4} e^{-\frac{\Delta E}{2T}} \frac{\alpha_{em}^2}{m_e^2} \sim \frac{T^2}{m_{Pl}}$$

$n_B \sim 10^{-10} T^3$

$$\Rightarrow \quad \frac{10^{-5}\, T^{-1/4} \alpha_{em}^2 m_{Pl}}{m_e^{5/4}} \sim e^{\frac{\Delta E}{2T}}$$

$$\Leftrightarrow \quad T = \frac{\Delta E}{2} \cdot \frac{1}{\ln\left(\frac{T^{-1/4} m_{Pl}}{10^9\, m_e^{5/4}}\right)} .$$

We may solve this iteratively, setting $T \sim \mathrm{eV}$ inside the logarithm.

$$\Rightarrow \quad T \approx \frac{6.8\,\mathrm{eV}}{\ln\left(\frac{(10^{-9}\,\mathrm{GeV})^{-1/4}\, 10^{19}\,\mathrm{GeV}}{10^9\,(10^{-3}\,\mathrm{GeV})^{5/4}}\right)} \approx \frac{6.8\,\mathrm{eV}}{\ln\left(10^{10-\frac{2}{4}+\frac{15}{4}}\right)}$$

$$\approx \frac{6.8\,\mathrm{eV}}{\ln(10^{11.5})} = \frac{6.8\,\mathrm{eV}}{11.5\ln(10)} = \frac{6.8\,\mathrm{eV}}{26.5} = 0.26\,\mathrm{eV} \quad (3000\,\mathrm{K}) .$$

In spite of several rough approximations, this happens to be close to the correct temperature at which decoupling happens.

<u>Summary:</u> Even if an almost perfect black body spectrum is observed in the cosmic microwave background today, the photons haven't scattered since 13×10^9 years, and are „out-of-equilibrium."

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21 cm line

Recall the hyperfine splitting of the hydrogen spectrum: in the 1s ground state, electron and proton spins can be antiparallel ($\uparrow\downarrow$) or parallel ($\uparrow\uparrow$). This yields the „smallest-energy excitation" of ground-state hydrogen. The energy difference is $\Delta E = 5.87 \times 10^{-6}\,\text{eV}$. Using $\text{eV} = 11600\,\text{K}$, ΔE is often expressed as $T_* \equiv \Delta E = 0.068\,\text{K}$. In vacuum the decay rate is $\Gamma = 2.85 \times 10^{-15}\,\frac{1}{s} \approx \frac{1}{10^7\,y}$, and the wavelength of the photons emitted is $\lambda = 21.1\,\text{cm}$. Because Γ is small, the peak is very narrow.

(i) would-be equilibrium solution

Given the spin degeneracy $g_s = 2s+1$, the equilibrium abundances of the $s=1$ and $s=0$ states are related by

$$\left(\frac{n_1}{n_0}\right)_{eq} = 3\,e^{-\frac{\Delta E}{T}}.$$

The non-equilibrium distribution is usually parametrized with the help of a „spin temperature" T_s, as

$$\left(\frac{n_1}{n_0}\right) \equiv 3\,e^{-\frac{\Delta E}{T_s}}.$$

(ii) interaction rate

Energetic CMB photons may cause transitions which keep the system in equilibrium. But in addition, at $T \lesssim 10^{-3}\,\text{eV}$, the first structures start to form, and are surrounded by hydrogen clouds. The hydrogen can be excited by collisions, and then emit „Lyman-α" photons ($n=2 \to n=1$). The Lyman-α photons can in turn excite the spin transition („Lyman-α pumping").

More precisely, there are now several rates:

$$\frac{dY_0}{dx} = -\hat{\Gamma}_{00}(Y_0 - Y_{0,eq}) - \hat{\Gamma}_{01}(Y_1 - Y_{1,eq}),$$

$$\frac{dY_1}{dx} = -\hat{\Gamma}_{10}(Y_0 - Y_{0,eq}) - \hat{\Gamma}_{11}(Y_1 - Y_{1,eq}).$$

Here $Y_i := \frac{n_i}{s}$, cf. p. 14. See, e.g., A. Loeb and M. Zaldarriaga, astro-ph/0312134

Conclusion: 21cm transitions are sensitive to the earliest stages of non-linear structure formation!

(iii) decoupling from equilibrium

As long as the system is in equilibrium, emission and absorption cancel against each other, and no spectral line is visible.

Once the system falls out of equilibrium, either an emission or absorption line can be seen, superimposed on the CMB.

Comparing with simulations, this should allow to extract information about the early stages of structure formation and, in particular, about the nature of the dark matter that plays an essential role in that dynamics.

In March 2018 the EDGES radio telescope announced the first observation of an „absorption profile" that could be the 21cm line. Many new experiments will follow in the next years and decades.

Nucleosynthesis

Big bang nucleosynthesis is a cornerstone of cosmology, but complicated to treat (weak, electromagnetic and strong interactions all play a role!), so we'll be brief.

(i) would-be equilibrium solution

Compared with the density of hydrogen atoms, we have $\left(\frac{n_N}{n_H}\right)_{eq} = \frac{g_N}{g_H} e^{-\frac{m_N - m_H}{T}}$.

For protons and neutrons, with $g_n = g_p = 2$, $\left(\frac{n_n}{n_p}\right)_{eq} = e^{-\frac{1.29\,\text{MeV}}{T}}$.

(ii) interaction rates

The general evolution is described by rate equations of the type $(Y_i := \frac{n_i}{s})$

$$\frac{dY_i}{dx} = -\sum_j \hat{\Gamma}_{ij} (Y_j - Y_{j,eq}).$$

At early times, the most important reactions are conversions $n \leftrightarrow p$:

$\nu_e + n \leftrightarrow p + e^-$ } weak interaction, like on p. 6: $\Gamma \sim G_F^2 T^5$.

$n \to p + e^- + \bar{\nu}_e$ } vacuum decay: $\Gamma \sim \frac{1}{880\,s}$.

(iii) decoupling from equilibrium

Similarly to p. 6, weak interactions fall out of equilibrium, now at $T \sim 0.8\,\text{MeV}$. So we may expect $\frac{n_n}{n_p} \sim e^{\frac{-1.29}{0.8}} \sim \frac{1}{6}$. Afterwards, some neutrons still decay, so that $\frac{n_n}{n_p} \sim \frac{1}{7}$ by the time that $T \sim 0.01\,\text{MeV}$.

For the other reactions, an important feature is the so-called deuterium bottleneck. Since there are many photons compared to baryons ($\frac{n_B}{n_\gamma} \sim 6 \times 10^{-10}$, cf. below), the reaction $d + \gamma \leftrightarrow p + n$ leads to efficient photodisintegration, and the formation of heavy elements is delayed, until $T < 0.1\,\text{MeV}$.

Helium abundance: Once $T < 0.1\,\text{MeV}$, most neutrons go into ^{4}He. So we can now estimate its mass fraction:

$$\frac{e_{He}}{e_B} \approx \frac{4\, n_{He}}{n_n + n_p} \approx \frac{4(n_n/2)}{n_n + n_p} = \frac{2\frac{n_n}{n_p}}{1 + \frac{n_n}{n_p}} \approx \frac{2\,\frac{1}{7}}{1 + \frac{1}{7}} = 0.25.$$

Other elements are only formed in small quantities: