Cosmological neutrino background

In order to illustrate with a relatively simple example how particles behave in the early universe, we start by establishing how neutrinos „decouple" at $T \approx 2$ MeV.

Neutrino basics

Three light neutrino flavours have been discovered. There is $\mathcal{O}(1)$ mixing between weak interaction and mass eigenstates ($\theta_{12} \approx 34^\circ$, $\theta_{23} \approx 50^\circ$, $\theta_{13} \approx 9^\circ$). Two mass differences are known ($\Delta m^2_{sol} \approx 7\times 10^{-5}\,\mathrm{eV}^2$, $\Delta m^2_{atm} \approx 3\times 10^{-3}\,\mathrm{eV}^2$). Open issues are absolute mass scale, mass ordering, and whether light neutrinos are Majorana- or Dirac-like.

(4 states like $e^\pm$ but only two are „excited" in practice)

Equilibrium solution

We might think that neutrinos of mass m_{ν_a}, $a \in \{e, \mu, \tau\}$, appear in the plasma with the phase space distribution

$$f(k) = n_F(\omega_a) := \frac{1}{e^{\omega_a/T}+1}\,, \quad \omega_a := \sqrt{k^2 + m^2_{\nu_a}}\,,$$

and thereby affect e, H, and everything that is going on.

Key point

As the universe is evolving, the temperature T changes. Therefore $n_F(\omega_a)$ changes. However, the only change that free particles experience is gravitational redshift. This does not necessarily conserve the equilibrium form. To find out, we must compute a scattering rate and see what the real-time solution looks like.

Mathematical formulation

Let us start with Minkowskian spacetime. With time, any system approaches equilibrium:

$$\dot{f} = -\Gamma(k)\left[f - n_F(\omega_a)\right] + \mathcal{O}\left[f - n_F(\omega_a)\right]^2$$

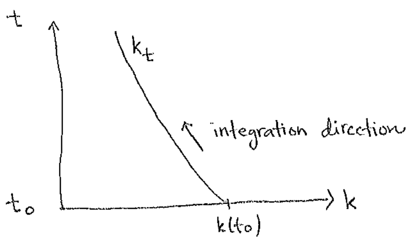
in covariant notation, $\partial_t \to u\cdot\partial$, where u is the plasma four-velocity

microscopic interaction rate from weak scatterings with plasma

Can be omitted if we are close to equilibrium („linear response regime")

In an expanding background this becomes

$$\left(\partial_t - Hk\partial_k\right) f(t,k) \approx -\Gamma(k)\left[f - n_F(\omega_a)\right].$$



„method of characteristics"

Ansatz: $\quad f(t,k) = f\left(t,\, k(t_0)\frac{a(t_0)}{a(t)}\right).$

Then $$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial k}\left(-k(t_0)\frac{a(t_0)}{a^2(t)}\dot{a}(t)\right) = \frac{\partial f}{\partial t} - kH\frac{\partial f}{\partial k}.$$

$$\Rightarrow \frac{df(t,k_t)}{dt} \approx -\Gamma(k_t)\left[f - n_F(\omega_a)\right], \quad k_t := k(t_0)\frac{a(t_0)}{a(t)}.$$

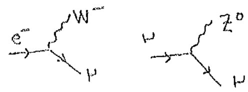
Let us furthermore go over to the coordinate $x \equiv \ln\left(\frac{T_{max}}{T}\right)$ and recall the Jacobian $J = \frac{dx}{dt} = 3c_s^2 H$ (p.3).
Measuring everything with T, we let $k_T := k(T_0)\frac{a(T_0)}{a(T)}$.
Now $\frac{df}{dt} = \frac{dx}{dt}\frac{df}{dx}$ and, dividing by J,

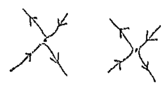
$$\boxed{\frac{df(x,k_T)}{dx} \approx -\hat{\Gamma}(k_T)\left[f - n_F(\omega_T)\right]} \quad ,$$

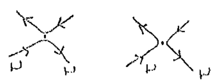
where we have denoted $\omega_T := \sqrt{k_T^2 + m_{\nu_a}^2}$ and $\hat{\Gamma} := \frac{\Gamma}{J}$.

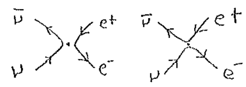
Summary: how efficiently systems follow equilibrium depends on the ratio of an interaction rate Γ and the Hubble rate H (assuming that $3c_s^2 \approx 1$).

Estimate of $\Gamma, \hat{\Gamma}$

Neutrinos feel only weak interactions: 

At low energies we can use the Fermi model .

Elastic scatterings $\leftrightarrow$ kinetic equilibration: 

Inelastic reactions $\leftrightarrow$ chemical equilibration: 

The vertex is proportional to the Fermi constant

$$G_F = \frac{g_w^2}{4\sqrt{2}\, m_w^2} \quad .$$

If $T \gg m_e \approx 0.5\,\text{MeV}$, then T is the only scale. So

$$\Gamma \sim \int d\Omega\, |M|^2 (n_F n_F - \ldots) \sim G_F^2 T^5 \quad .$$

The Hubble rate is $H \sim \frac{T^2}{m_{Pe}}$ (p.3).

$$\Rightarrow \hat{\Gamma} \sim m_{Pe}\, G_F^2\, T^3 \quad .$$

In general $\hat{\Gamma} \ll 1$ or $\hat{\Gamma} \gg 1$. When is $\hat{\Gamma} \sim 1$?

$$T \sim \left(\frac{1}{m_{Pe}\, G_F^2}\right)^{1/3} \sim \left(\frac{1}{10^{19}\,\text{GeV}\left(\frac{10^{-5}}{\text{GeV}^2}\right)^2}\right)^{1/3} \sim 10^{-3}\,\text{GeV} \, .$$

A more precise computation, including numerical prefactors, yields $\hat{\Gamma} \approx 1$ at $T \approx 2\,\text{MeV}$ for „typical" momenta $k \approx (1\ldots5)\,T$.

<u>Solution of evolution equation:</u> Let us inspect $\partial_x f \approx -\hat{\Gamma}[f - n_F(\omega_T)]$ in the so-called „sudden decoupling approximation". We also omit m_{ν_a} for simplicity.

* $\hat{\Gamma} \gtrsim 1$ until $x = x_1$ $\Rightarrow$ f stays close to n_F

$$\Rightarrow \quad f(x_1, k) \approx \frac{1}{e^{k/T_1} + 1} .$$

* $\hat{\Gamma} \ll 1$ afterwards $\Rightarrow$ right-hand side vanishes

$$\Rightarrow \quad f(x_2, k) \approx f\left(x_1, \frac{k\, a(T_2)}{a(T_1)}\right)$$

Now we can compute the energy density in one neutrino flavour:

$$e_\nu = 2 \int_{\vec{k}} k\, f(x_2, k) = 2 \int_{\vec{k}} \frac{k}{e^{\frac{k\, a(T_2)}{T_1 a(T_1)}} + 1}$$

$$= 2 \cdot \left[\frac{T_1 a(T_1)}{a(T_2)}\right]^4 \cdot \frac{7}{8} \frac{\pi^2}{30}$$

$$= 2 \left[\frac{T_1 a(T_1)}{T_2 a(T_2)}\right]^4 \cdot \frac{7}{8} \int_{\vec{k}} \frac{k}{e^{\frac{k}{T_2}} - 1}$$

$$= \frac{7}{8} \left[\frac{T_1^3 a^3(T_1)}{T_2^3 a^3(T_2)}\right]^{\frac{4}{3}} e_\gamma .$$

Let $T_2 \ll 1$ MeV, so that electrons have become non-relativistic.

Given that $s a^3$ is constant (p. 2), we can estimate

$$\frac{T_1^3 a^3(T_1)}{T_2^3 a^3(T_2)} = \frac{T_1^3}{s(T_1)} \cdot \frac{s(T_2)}{T_2^3} \cdot \underbrace{\frac{s(T_1) a^3(T_1)}{s(T_2) a^3(T_2)}}_{1} \overset{\text{like on p.3}}{\approx} \frac{2}{2 + \frac{7}{8} \cdot 4} = \frac{4}{11} .$$

So we parametrize the result as

$$\frac{e_\nu}{e_\gamma} = \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} N_{\rm eff} .$$

A more precise computation, including masses, neutrino oscillations, smooth decoupling, etc, yields*

$$N_{\rm eff} \simeq 3.045 .$$

* e.g. P.F. de Salas, S. Pastor, „Relic neutrino decoupling with flavour oscillations revisited", 1606.06986.

<u>Low temperatures</u>

Ultimately, when $T \ll$ eV, neutrinos become non-relativistic as a result of the cosmological redshift. However, their phase space distribution is still given by the solution discussed above, and therefore looks „relativistic". Let us estimate the energy density in this regime:

↘

$$e_\nu \approx 2\sum_a m_{\nu_a} \int_k \frac{1}{e^{\frac{k a(T_2)}{T_1 a(T_1)}}+1} = 2\sum_a m_{\nu_a}\left[\frac{T_1 a(T_1)}{a(T_2)}\right]^3 \frac{3}{4}\frac{\zeta(3)}{\pi^2}$$

$$= 2\sum_a m_{\nu_a}\left[\frac{T_1 a(T_1)}{T_2 a(T_2)}\right]^3 \frac{3}{4}\int_k \frac{1}{e^{k/T_2}-1} = \sum_a m_{\nu_a}\, \frac{T_1^3}{s(T_1)}\cdot\frac{s(T_2)}{T_2^3}\cdot\frac{3}{4}\cdot n_\gamma$$

Experimental constraints:

The energy density carried by neutrinos affects H, and thereby rate coefficients $\sim\hat{\Gamma}$ normalized by H, and therefore many phenomena in the late universe that we will discuss later on, like BBN ($T\sim 0.1\,\text{MeV}$), CMB ($T\sim 0.1\,\text{eV}$), and LSS ($T \ll 0.1\,\text{eV}$). Moreover, neutrinos affect the non-linear growth of density perturbations in a somewhat complicated way.

$\Rightarrow$ CMB data from Planck* yields $N_{\text{eff}} \simeq 2.96 \pm 0.34$

$\Rightarrow$ CMB + LSS data yields $\sum_a m_{\nu_a} < 0.12 \ldots 0.73\,\text{eV}$

$\Rightarrow$ fraction of energy density in neutrinos: $\Omega_\nu \lesssim 0.3\% \ll \Omega_{dm}$

* Planck Collaboration, „Planck 2018 results. VI. Cosmological parameters", 1807.06209

Of what type are neutrinos?

As mentioned at the beginning, neutrinos could be „Majorana-like" (2 light degrees of freedom per flavour) or „Dirac-like" (4 degrees of freedom per flavour)*.

* In general neutrinos have both a Majorana mass term M_M and a Dirac mass term M_D. The case $M_M \sim M_D$ would require fine tuning, so we normally focus on $M_M \gg M_D$ or $M_D \gg M_M$.

Q: Isn't the Dirac case excluded by $N_{\text{eff}} \simeq 3$?

A: Not really, because the „wrong-helicity" states would not be efficiently produced and therefore in practice never thermalize.

Production rate for states with mass-suppressed helicity:

$$\Gamma \overset{(?)}{\sim} G_F^2 m_\nu^2 T^3 .$$

Compare with Hubble rate (i.e. set $\hat{\Gamma}\sim 1$):

$$G_F^2 m_\nu^2 T^3 \sim \frac{T^2}{m_{Pl}}$$

$$\Rightarrow \quad T \sim \frac{1}{G_F^2 m_\nu^2 m_{Pl}} \sim \frac{1}{\left(\frac{10^{-5}}{\text{GeV}^2}\right)^2 (10^{-10}\,\text{GeV})^2\, 10^{19}\,\text{GeV}} \sim 10^{11}\,\text{GeV}.$$

But then Fermi model isn't valid, and the estimate needs to be refined $\Rightarrow$ conclusion stays.**

** e.g. J. Lesgourgues and S. Pastor, „Neutrino Cosmology and Planck", 1404.1740