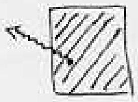
9.4 Particle production rates



* In cosmology: production rate for weakly interacting dark matter candidates, such as axions, axinos, gravitinos, ...

* In heavy ion collisions: production rate for real (on-shell) photons, or off-shell photons → lepton pairs ($\mu^+\mu^-$).

We start by considering the production rate for a scalar particle ϕ, and generalise then to real photons / lepton pairs.

Recall the basic rules:

* Asymptotic free scalar field:

$$\hat{\phi}(t,\bar{x}) = \int \frac{d^3k}{\sqrt{(2\pi)^3\, 2E_{\bar{k}}}} \left[\hat{a}_k e^{-ik\cdot x} + \hat{a}_k^{+} e^{ik\cdot x} \right] \quad , \quad x=(t,\bar{x}),\; k_0 = E_{\bar{k}} = \sqrt{\bar{k}^2+m^2}$$

Normalisation chosen so that

$$\hat{H}_0 = \int d^3k\, E_{\bar{k}}\, \hat{a}_k^{+}\hat{a}_k \quad , \quad [\hat{a}_k, \hat{a}_{k'}^{+}] = \delta^{(3)}(\bar{k}-\bar{k}') \quad ; \quad [\hat{a}_k] = GeV^{-3/2}$$

* Consider an arbitrary initial state $|i\rangle$, and a final state $|F\rangle \equiv |f\rangle \otimes |Q\rangle$, with $|Q\rangle$ a one-particle state with a free ϕ of momentum Q.

* Interaction Hamiltonian: $\hat{H}_I = \int d^3x\, \hat{\phi}(t,\bar{x})\, \hat{J}(t,\bar{x})$

* Transition matrix element to first order in $\hat{H}_I$:

$$T_{Fi} = \langle f| \otimes \langle Q| \int dt\, \hat{H}_I(t)\, |i\rangle$$

$$= \int dt \int d^3x\, \frac{1}{\sqrt{(2\pi)^3\, 2E_{\bar{q}}}}\, e^{iQ\cdot x} \langle f|\, \hat{J}(t,\bar{x})\, |i\rangle$$

$\Rightarrow$ Rate of transitions $\left(\Gamma = \frac{dN}{dt}\right)$ per volume $\left(\frac{\Gamma}{V}\right)$ to a phase space element $d^3\bar{q}$ around Q:

$$\frac{\Gamma}{V}(Q) \equiv \frac{dN(Q)}{d^4x} = \lim_{t,V\to\infty} \frac{1}{Vt} \, |T_{fi}|^2 \, d^3\bar{q}$$

Furthermore we have to sum now over all initial states $|i\rangle$ with their proper Boltzmann weights $\frac{1}{Z}\exp(-\beta E_i)$, as well as all possible final states

$$\Rightarrow \quad \frac{dN(Q)}{d^4x \; d^3\bar{q}} = \lim_{t,V\to\infty} \frac{1}{Vt} \frac{1}{(2\pi)^3 \cdot 2E_{\bar{q}}} \cdot \int dt\, d^3x \int dt'\, d^3x' \; e^{iQ\cdot(x-x')}$$

$$\times \underbrace{\sum_{f,i} \langle f | \hat{J}(t,\bar{x}) | i \rangle \, e^{-\beta E_i} \langle i | \hat{J}^+(t',\bar{x}') | f \rangle \cdot \frac{1}{Z}}_{\mathrm{Tr}\left[e^{-\beta \hat{H}} \hat{J}^+(t',\bar{x}')\, \hat{J}(t,\bar{x}) \right] \cdot \frac{1}{Z}}$$

The trace can only depend on $x - x'$, due to translational invariance

$$\Rightarrow \quad \frac{dN(Q)}{d^4x \; d^3\bar{q}} = \frac{1}{(2\pi)^3 \, 2E_{\bar{q}}} \int dt\, d^3x \; e^{iQ\cdot x} \left\langle \hat{J}^+(0,\bar{0})\, \hat{J}(t,\bar{x}) \right\rangle$$

This is precisely of the type on p. 105, 112!

$$\Rightarrow \quad \boxed{\frac{dN(Q)}{d^4x\, d^3\bar{q}} = \frac{d\,\Gamma/V}{d^3\bar{q}} = \frac{1}{(2\pi)^3\, 2q_0} \, \Pi^<(q_0, \bar{q})}$$

Recall also (p. 106): $\quad \Pi^<(q_0,\bar{q}) = 2\, n_b(q_0)\, \rho(q_0,\bar{q})$.

Generalizations

[McLerran, Toimela, Phys. Rev. D 31 (1985) 545 ; Gale, Kapusta, Nucl. Phys. B 357 (1991) 65
Weldon, Phys. Rev. D 42 (1990) 2384]

(i) Real photon production rate ("How does a plasma shine?")

* The electromagnetic current plays the role of $\hat{J}$:

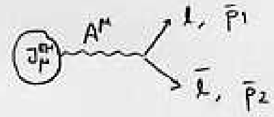
$$J_\mu^{EM}(x) = \frac{\partial \mathcal{L}^M}{\partial A^\mu_{EM}(x)} = e\left[\frac{2}{3}\bar{u}_\alpha\gamma_\mu u_\alpha - \frac{1}{3}\bar{d}_\alpha\gamma_\mu d_\alpha - \bar{l}\gamma_\mu l + \ldots\right]$$

* Sum over photon polarizations:

$$\sum_\lambda \varepsilon^\mu_{(\lambda)}\varepsilon^\nu_{(\lambda)} = -g^{\mu\nu}$$

$$\Rightarrow \quad \frac{dN_\gamma(Q)}{d^4x\, d^3\bar{q}} = -\frac{1}{(2\pi)^3\, 2q_0}\left[\Pi^<\right]^\mu{}_\mu(q_0, \bar{q})\Big|_{q_0 = |\bar{q}|}.$$

(ii) Dilepton production rate.



* First, integrate out A^μ_{EM}, to get

$$S_M^{eff} \sim \int_{x,y} e\bar{l}\gamma_\mu l\,(y)\; J_\nu^{EM}(x)\cdot \int_Q \frac{e^{iQ\cdot(x-y)}}{Q^2}\cdot g^{\mu\nu}$$

* Write $l, \bar{l}$ as free on-shell fields; pick up final state with momenta $\bar{p}_1, \bar{p}_2$

$$\Rightarrow T_{fi} \sim \frac{e}{Q^2}\bar{u}(p_1)\gamma^\mu v(p_2)\frac{1}{\sqrt{(2\pi)^3 2E_1}}\cdot\frac{1}{\sqrt{(2\pi)^3 2E_2}}\cdot$$

$$\times \int d^4x\, e^{iQ\cdot x}\langle f|J_\mu^{EM}(x)|i\rangle \Big|_{Q=(E_1+E_2,\, \bar{p}_1+\bar{p}_2)}$$

* Take $|T_{fi}|^2$. Sum over spin states,

$$\sum_{spins}\bar{u}(\bar{p}_1)\gamma^\mu v(\bar{p}_2)\cdot\left[\bar{u}(\bar{p}_1)\gamma^\nu v(\bar{p}_2)\right]^* = \ldots$$

* Rather than $\bar{p}_1, \bar{p}_2$, keep Q as the phase-space variable

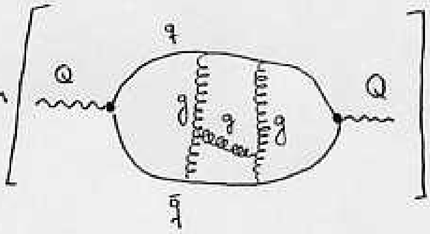
$$\Rightarrow \frac{dN(Q)}{d^4x\, d^4Q} = \frac{e^2}{(2\pi)^4 Q^4}\cdot L^{\mu\nu}(Q)\cdot \Pi^<_{\mu\nu}(Q),$$

$$L^{\mu\nu}(Q) = \frac{1}{6\pi}\cdot\left(1+\frac{2m^2}{Q^2}\right)\left(1-\frac{4m^2}{Q^2}\right)^{1/2}\left(Q^\mu Q^\nu - Q^2 g^{\mu\nu}\right).$$

A few remarks on the computation of $\Pi^<_{\mu\nu}(q_0, \bar{q})$

(A) Diagrams

$\Pi^< \propto \rho \propto \mathrm{Im}$



real photons: $Q^2 = 0$ (on-shell)

dileptons: $Q^2 \geq (2m_\ell)^2$ (off-shell)

— These computations are in general very subtle and complicated, even as far as the leading order result is concerned. Not only does one need HTL-resummations, but also others ("ladder" etc). This is true particularly for $Q^2 \approx 0$, the on-shell point. Nevertheless, much progress in recent years.

— Somewhat easier "off-shell", e.g. $q_0 \gtrsim T$, $\bar{q} = 0$ (purely perturbative), or if we consider the contribution of heavy quarks ($m_q \gg T$).

(B) Lattice

Because of the complications mentioned above, and because in realistic systems the coupling is not small, there would be a need for a non-perturbative determination of the spectral function for J^{EM}_μ. There are, however, rather serious fundamental problems here, related to analytic continuation of numerical data (invert the relation on p. 113 !)

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