9. Applications

Thermal field theory has physical applications in at least three major contexts:

(i) Early Universe cosmology.

(ii) Heavy Ion Collision experiments, which try to imitate (i).
("Little Bang" vs "Big Bang")

(iii) Cores of dense astrophysical objects like neutron stars; low temperature but very high baryon density.

In the following we concentrate on (i), although many things apply to (ii) as well.

9.1. The cosmological background

What is our Universe made of?

Underlying theory	Almost stable particle/field	Fraction of present energy density	Universal characteristic
QCD	p^+, n inside H, D, He, Li, ...	$\sim 5\%$	Baryon number B
EW	e^-	$\sim 0\%$	Lepton number L
	ν_e, ν_μ, ν_τ	$\sim \%$?	Lepton number L
	γ – in background radiation	$\sim 0\%$	
	γ – in classical magnetic fields	$\sim \%$?	
Supersymmetry	LSP	$\sim 25\%$?	"R-parity"
Gravity	Cosmological constant	$\sim 70\%$	

Another relic is a spatial fluctuation in these.

The challenge is to understand how these number and their spatial distributions come about.

How would the Universe look like in full equilibrium?

* Utterly dull. Homogeneous, only characterised by T, μ_B, μ_L, μ_R. No light elements, all B in iron. In fact no B, as after inflation $B = L = R = 0$, and in equilibrium nothing can be generated
  $\Rightarrow$ need necessarily deviation from equilibrium!

How to deviate from equilibrium?

Let us introduce two different time scales:

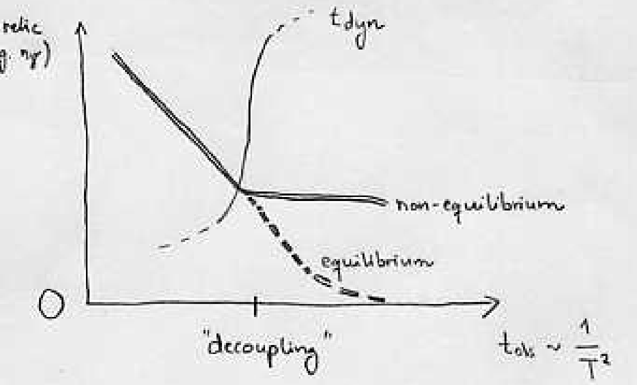
t_{dyn} = inherent dynamical time scale
t_{obs} = external observation time scale

(Think, for instance, of the Boltzmann equation in the relaxation time approximation:

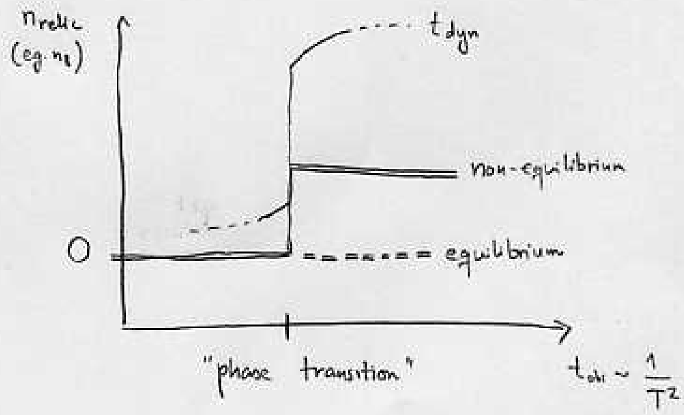
$$\frac{\partial f(\bar{x}, \bar{p}, t_{obs})}{\partial t_{obs}} = -\frac{1}{t_{dyn}} \left[f(\bar{x}, \bar{p}, t_{obs}) - f_0(T, \mu_i) \right].$$
)

Two ways to deviate from equilibrium:

① "Smooth"



② "Abrupt"



How to get an order of magnitude estimate of t_{dyn}, t_{obs}?

(A) t_{dyn} comes from microscopic particle physics: in kinetic theory,

$$t_{dyn} \sim \frac{1}{n\sigma v} \quad ; \quad \begin{array}{l} n = \text{density} \\ \sigma = \text{cross section} \\ v = \text{velocity} \approx 1 . \end{array}$$

(B) t_{obs} comes from macroscopic cosmology:

Metric: $ds^2 = dt^2 - a^2(t)\, d\vec{x}_i^2$

Ideal fluid: $T_\mu{}^\nu = \mathrm{diag}(e, -p, -p, -p)$, e = energy density, p = pressure

Einstein: $G_\mu{}^\nu = 8\pi G\, T_\mu{}^\nu$

$$\Rightarrow \begin{cases} \left(\frac{\dot a}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3} e \\ d(ea^3) = -p\, d(a^3) \end{cases} \qquad (*)$$

Assume flat Universe, $k=0$, and denote

$$G \equiv \frac{1}{m_{Pl}^2} \quad , \quad m_{Pl} = 1.2 \times 10^{19}\ \mathrm{GeV} .$$

Using $s \equiv \frac{dp}{dT}$, $e = Ts - p$ (for $\mu = 0$), eliminate $a(t)$ from $(*)$

$$\Rightarrow \quad \frac{dT}{dt} = -\frac{\sqrt{24\pi}}{m_{Pl}} \cdot \frac{\sqrt{e(T)}}{d[\ln s(T)]/dT}$$

For ideal gas, $p(T) = -f(T) = g_* \frac{\pi^2}{90} T^4$,

$$g_* = \sum_{m_b \ll T} + \frac{7}{8} \sum_{m_f \ll T}$$

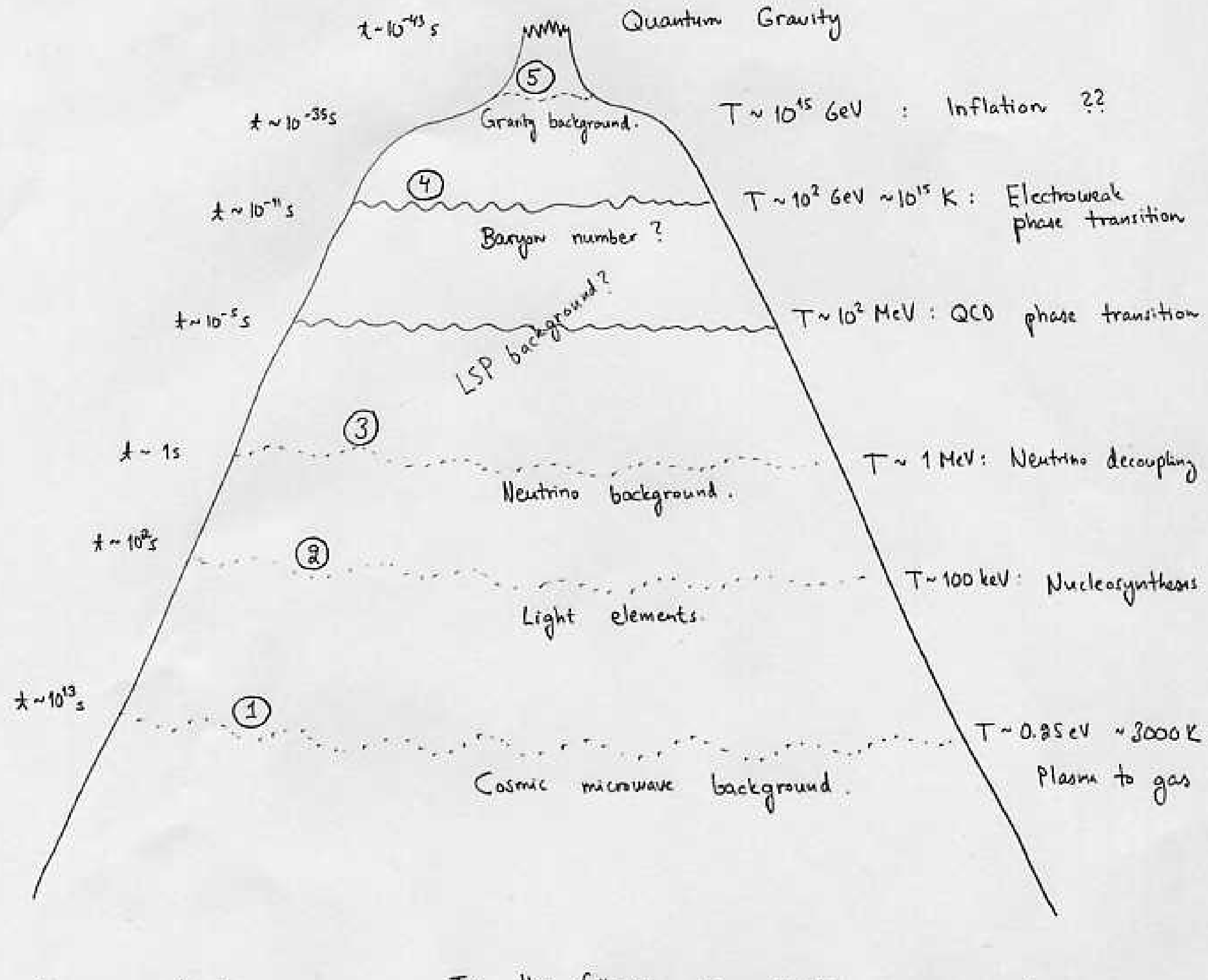
$$\Rightarrow \quad \boxed{t = \frac{3}{4}\sqrt{\frac{5}{\pi^3 g_*}}\, \frac{m_{Pl}}{T^2} \quad ; \quad \frac{t}{\mathrm{sec}} \sim \left(\frac{\mathrm{MeV}}{T}\right)^2 .}$$

Non-equilibrium if $t_{dyn} \sim \frac{1}{n\sigma} \gg t_{obs} \sim \frac{m_{Pl}}{T^2}$.

Basic processes:

1. <u>Photons</u>: $e+p \leftrightarrow H+\gamma$, $e+\gamma \leftrightarrow e+\gamma$ $\Rightarrow$ when $T \ll eV$, n_e small, and $t_{dyn} \to \infty$.
2. <u>Light elements</u>: $e+p \leftrightarrow n+\nu$, $p+n \leftrightarrow d+\gamma$, etc $\Rightarrow$ when $T \ll M_n - m_p \sim$ MeV, n_n is small, and $t_{dyn} \to \infty$.
3. <u>Neutrinos</u>: $\sigma \sim \frac{\alpha_w^2 T^2}{m_w^4}$, $n \sim T^3$ $\Rightarrow$ $t_{dyn} \gg t_{obs}$ for $T \ll \left(\frac{m_w^4}{\alpha_w^2 m_{Pl}}\right)^{1/3} \sim$ MeV.
4. <u>Baryon number</u>: $\frac{1}{t_{dyn}} \sim T \exp\left(-\frac{45|\bar{\phi}|}{T}\right)$ $\Rightarrow$ $t_{dyn} \gg t_{obs}$ for $|\bar{\phi}| \gtrsim T$.
5. <u>QCD</u>: $\sigma \sim \frac{\alpha_s^2}{T^2}$, $n \sim T^3$ $\Rightarrow$ $t_{dyn} \gg t_{obs}$ for $T \gg \alpha_s^2 m_{Pl} \sim 10^{15}$ GeV !

Etc.



~~~~ = abrupt

.... = smooth

In the following we consider more precisely:

(i) existence of phase transitions;

(ii) nucleation rate through phase transitions;

(iii) production rate for weakly interacting particles (LSP).

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