8.1 A sample computation

Let us now apply the relations found to a practical computation. Two possible formulations:

(i) "Imaginary time followed by analytic continuation".

$\Rightarrow$ Start by computing the "easy" observables $\Pi_E(\omega_n)$ | $\tilde{\Pi}_E(\tau)$ in the imaginary time (Matsubara) formalism. Then apply the relations obtained to this result, to get $\rho(p_0)$.

(ii) "Real-time formalism"

$\Rightarrow$ Formulate the analytic continuation on the level of Feynman rules already. Naively this means carrying out computations as at zero temperature in Minkowskian spacetime, but with a thermal correction in the propagator (p 108):

$$-i\Pi_T(p_0) = \underbrace{\frac{+i}{p_0^2 - E^2 + i\epsilon}}_{\text{as at } T=0} + \underbrace{2\pi\,\delta(p_0^2 - E^2)\, n_b(|p_0|)}_{\text{temperature}}$$

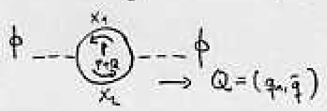
Unfortunately a more careful analysis reveals that this shortcut does not work in general: one needs to introduce several propagators, corresponding to $\Pi^>$, $\Pi^<$, ...

[see the book by Le Bellac, for instance]

People tend to have (strong) personal preferences as to which way is better. In the following we use (i).

Let us now consider the following example: [H.A. Weldon, Phys. Rev. D 28 (1983) 2007]

$$\mathcal{L}_I \sim g\phi\chi_1\chi_2$$



$$\Pi_E(q_n,\bar{q}) \equiv g^2\, T\sum_{p_n}\int\frac{d^3\bar{p}}{(2\pi)^3}\,\frac{1}{[p_n^2+p_0^2][(p_n+q_n)^2+r_0^2]} \quad ; \quad \begin{array}{l} p_0^2 \equiv E_1^2 \equiv m_1^2+\bar{p}^2 \\ r_0^2 \equiv E_2^2 \equiv m_2^2+(\bar{p}+\bar{q})^2 \end{array}$$

(1) Carry out $T\sum_{p_n}$, using

$$T\sum_{p_n} f(p_n) = \frac{1}{2}\int_{-\infty-i\varepsilon}^{\infty-i\varepsilon}\frac{d\omega}{2\pi}\,\big[f(\omega)+f(-\omega)\big]\big[1+2n_b(i\omega)\big] .$$

Factor out $g^2\int\frac{d^3\bar{p}}{(2\pi)^3}$ for a moment.

Write:

$$f(\omega)+f(-\omega) = \frac{1}{(\omega^2+p_0^2)}\left[\frac{1}{(\omega+q_n)^2+r_0^2}+\frac{1}{(-\omega+q_n)^2+r_0^2}\right]$$

$$= \frac{1}{(2p_0)(2r_0)}\left[\frac{i}{\omega+ip_0}-\frac{i}{\omega-ip_0}\right]$$
$$\times\left[\frac{i}{\omega+q_n+ir_0}-\frac{i}{\omega+q_n-ir_0}+\frac{i}{-\omega+q_n+ir_0}-\frac{i}{-\omega+q_n-ir_0}\right]$$

Pick up poles in the <u>lower half plane</u>:

$$\frac{1}{2}\cdot\frac{1}{2\pi}\cdot(-2\pi i)\cdot\frac{i}{(2p_0)(2r_0)}\Bigg\{[\omega=-ip_0]\;(1+2n_b(p_0))\left(\frac{i}{q_n-ip_0+ir_0}-\frac{i}{q_n-ip_0-ir_0}+\frac{i}{q_n+ip_0+ir_0}-\frac{i}{q_n+ip_0-ir_0}\right)$$

$$[\omega=-q_n-ir_0]\;(1+2n_b(r_0))\left(-\frac{i}{q_n-ip_0+ir_0}+\frac{i}{q_n+ip_0+ir_0}\right)$$

$$[\omega=q_n-ir_0]\;(1+2n_b(r_0))\left(\frac{i}{q_n+ip_0-ir_0}-\frac{i}{q_n-ip_0-ir_0}\right)\Bigg\}$$

$$\boxed{\text{Note: } n_b(\mp iq_n+r_0) = n_b(r_0)\ !!}$$

$$= \frac{1}{(2p_0)(2r_0)} \left\{ \frac{1}{-iq_n - p_0 + r_0} \left[\cancel{\tfrac{1}{2}} + n_b(p_0) - \cancel{\tfrac{1}{2}} - n_b(r_0) \right] \right.$$

$$+ \frac{1}{iq_n + p_0 + r_0} \left[\tfrac{1}{2} + n_b(p_0) + \tfrac{1}{2} + n_b(r_0) \right]$$

$$+ \frac{1}{-iq_n + p_0 + r_0} \left[\tfrac{1}{2} + n_b(p_0) + \tfrac{1}{2} + n_b(r_0) \right]$$

$$\left. + \frac{1}{iq_n - p_0 + r_0} \left[\cancel{\tfrac{1}{2}} + n_b(p_0) - \cancel{\tfrac{1}{2}} - n_b(r_0) \right] \right\}$$

Now: $\quad * \quad \rho(q_0) = \operatorname{Im} \Pi_E \Big|_{iq_n \to q_0 + i\varepsilon}$

$$* \quad \operatorname{Im} \frac{1}{x \pm i\varepsilon} = \mp \pi \delta(x)$$

$$\Rightarrow \quad \rho(q_0) = g^2 \pi \int \frac{d^3\vec{p}}{(2\pi)^3} \cdot \frac{1}{(2p_0)(2r_0)} \cdot \left\{ \delta(-q_0 - p_0 + r_0) \left[n_b(p_0) - n_b(r_0) \right] \right.$$
$$- \delta(q_0 + p_0 + r_0) \left[1 + n_b(p_0) + n_b(r_0) \right]$$
$$+ \delta(-q_0 + p_0 + r_0) \left[1 + n_b(p_0) + n_b(r_0) \right]$$
$$\left. - \delta(q_0 - p_0 + r_0) \left[n_b(p_0) - n_b(r_0) \right] \right\} \qquad (*)$$

Or, denoting $\quad p_0 = E_1, \quad n_b(p_0) = n_b(E_1) \equiv n_1$

$\qquad\qquad\quad r_0 = E_2, \quad n_b(r_0) = n_b(E_2) \equiv n_2$:

$$\rho(q_0) = g^2 \pi \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{(2E_1)(2E_2)} \left\{ \delta(q_0 + E_1 + E_2) \left[n_1 n_2 - (1+n_1)(1+n_2) \right] \right.$$
$$+ \delta(q_0 + E_1 - E_2) \left[n_1(1+n_2) - (1+n_1) n_2 \right]$$
$$+ \delta(q_0 - E_1 + E_2) \left[n_2(1+n_1) - (1+n_2) n_1 \right]$$
$$\left. + \delta(q_0 - E_1 - E_2) \left[(1+n_1)(1+n_2) - n_1 n_2 \right] \right\} \qquad (**)$$

Physical interpretations

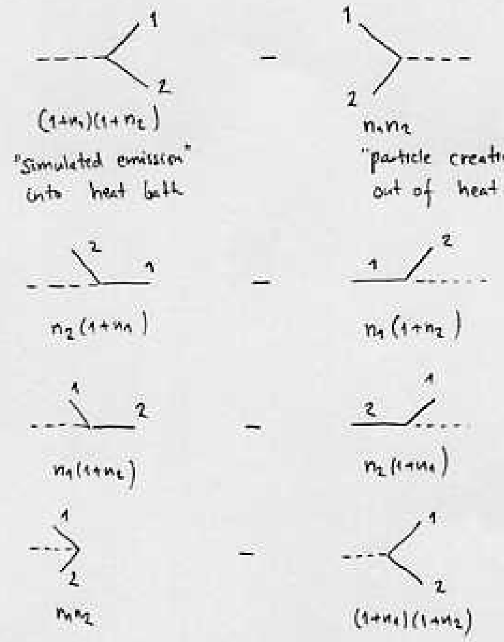
Let us inspect eq. (**) on p. 111. It represents the following processes:

$\delta(q_0 - E_1 - E_2)$

$\delta(q_0 - E_1 + E_2)$

$\delta(q_0 + E_1 - E_2)$

$\delta(q_0 + E_1 + E_2)$



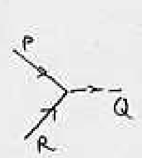
$\Rightarrow \quad \rho(q_0, \bar{q}) \sim$ disappearance rate of particles with energy q_0, momentum $\bar{q}$?

Actually it can be shown (see below) that the rate at which ϕ-particles are produced and <u>escape</u> from the plasma, is

$$\frac{d[\Gamma/V]}{d\bar{q}^3} = \frac{1}{(2\pi)^3}\frac{1}{(2q_0)}\, \Pi^<(q_0, \bar{q}) = \frac{1}{(2\pi)^3}\cdot\frac{1}{(2q_0)}\cdot 2n_b(q_0)\,\rho(q_0, \bar{q}) \,.$$

The outcome is that at 1-loop order, the formalism of finite-temperature field theory just reproduces the rate that would be guessed based on classical Boltzmann equations!

$\Rightarrow$ Using $n_b(q_0)\left[1 + n_b(p_0) + n_b(r_0)\right]\delta(q_0 - p_0 - r_0) = n_b(p_0)\, n_b(r_0)\, \delta(q_0 - p_0 - r_0)$,



$$\frac{\Gamma}{V} = \int \frac{d^3q}{(2\pi)^3(2q_0)} \frac{d^3p}{(2\pi)^3(2p_0)} \frac{d^3r}{(2\pi)^3(2r_0)} (2\pi)^4 \delta^{(4)}(Q-P-R)\, |g|^2\, n_b(p_0) n_b(r_0) \cancel{(1 + n_b(q_0))} + \ldots$$

missing, since the particles produced are assumed <u>to escape</u>!

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