Exercise 1: *Schur lemma:* let $\mathscr{D}(g)$ be the representation matrices of an irreducible representation, and S a matrix with the property $\mathscr{D}(g)S = S\mathscr{D}(g) \quad \forall g \in G$. Show that S is necessarily proportional to the unit matrix.

[*Hint:* Irreducibility implies that there are no invariant subspaces. Let v_λ be an eigenvector of S, i.e. $Sv_\lambda = \lambda v_\lambda$. Show that then also $\mathscr{D}(g)v_\lambda$ is an eigenvektor of S, with the same eigenvalue. Because there are no invariant subspaces, the orbit $\{\mathscr{D}(g)v_\lambda\}$ must span the full representation space. Therefore, for **all** vectors of the representation space, $Sv = \lambda v$. Show that this is only possible if S is proportional to the unit matrix.]

Exercise 2: What is the center of SU(n)? [*Hint:* Make use of the Schur lemma.]

Exercise 3: In the lecture we have defined the homomorphism $f : SL(2,\mathbb{C}) \to L_+^\uparrow$, $A \mapsto \Lambda(A) \in L_+^\uparrow$, which acts as

$$M' \equiv x'^\mu \, \sigma_\mu \equiv \Lambda(A)^{\,\mu}{}_{\nu} \, \underbrace{x^\nu \sigma_\mu}_{\equiv M} \equiv A \, M \, A^\dagger \, .$$

(a) What is ker(f)?
[*Hint:* If $A \in$ ker(f), then we must have $M = A\,M\,A^\dagger$ for all Hermitean matrices M. By choosing $M = I$, it follows that $A^\dagger = A^{-1}$. Then one may employ arguments similar to those in the Schur lemma.]

(b) To which quotient group is $L_+^\uparrow$ isomorphic?

Exercise 4: We know that SU(2) and SO(3) are homomorphic to each other (cf. Exercise 1.4). Therefore they have the same Lie algebras. Show that their centers are, however, different.
[*Remark:* Ultimately this fact excludes half integer representations from SO(3). SU(2) is the "universal covering group" of SO(3), and we have the isomorphism SU(2)/$\mathbb{Z}_2 \cong$ SO(3).]