

Exercise 1: Consider the groups $SU(2)$ and $SU(3)$.

- (a) What is the fundamental weight of $SU(2)$?
- (b) Convince yourself that there is a representation with each dimension $d = 1, 2, \dots$, and in each case only one per a given d .
- (c) For a representation (q^1, q^2) of $SU(3)$, the dimension is $d(q^1, q^2) = \frac{(q^1+1)(q^2+1)(q^1+q^2+2)}{2}$. Are there representations of each dimension $d = 1, 2, \dots$?
- (d) Are there independent $SU(3)$ representations with the same d ?

Exercise 2: The u and d quarks have (provided that electromagnetic effects can be ignored) almost the same properties. This fact motivates the introduction of an (approximate) $SU(2)$ invariance, known as the isospin symmetry. We can identify the u and d quarks as the states $|1/2, 1/2\rangle$, $|1/2, -1/2\rangle$, and the antiquarks \bar{u} , \bar{d} as $-|1/2, -1/2\rangle$, $|1/2, 1/2\rangle$. The pions are composed of a quark and an antiquark, and possess the isospin identifications $\pi^+ = |1, 1\rangle$, $\pi^0 = |1, 0\rangle$, $\pi^- = |1, -1\rangle$.

- (a) Write π^+ , π^0 and π^- as linear combinations of $u\bar{u}$, $u\bar{d}$, $d\bar{u}$, $d\bar{d}$.
- (b) Which isospin has the state $(u\bar{u} + d\bar{d})$?

[Hint: Clebsch-Gordan decomposition is reviewed in section 1.6.]

Exercise 3: Construct the weight diagram for the presentation $(3, 0)$ of $SU(3)$. [This diagram is famous in physics, because it led Gell-Mann to predict the existence of a yet unknown baryon, called Ω^- . It was soon discovered, and Gell-Mann got the 1969 Nobel prize for physics.]

Exercise 4: Consider the conjugated representation $\mathbf{3}^*$ of $SU(3)$, with the generators $-(T^a)^*$.

- (a) Show that the weights of $\mathbf{3}^*$ have the opposite sign to those of $\mathbf{3}$. Draw the weight diagram of $\mathbf{3}^*$.
- (b) Show that $\mathbf{3}^* = (0, 1)$.
- (c) Convince yourself that in general $(q^1, q^2)^* = (q^2, q^1)$. [Remark: The representations (n, n) are therefore invariant under $w^j \rightarrow -w^j$, and are called "real representations".]