ACTP	**sheet nr. 04**

Exercise 1: Let us consider the Lie algebra $su(2)$ in the fundamental representation $\mathbf{2}$.

(a) Construct the Cartan subalgebra **h**.

(b) What are the weight vectors?

(c) Construct the independent generators E_k with $[H_i, E_k] = \alpha_i^k E_k$ for $H_i \in \mathbf{h}$.

(d) What are the root vectors?

[*Remark:* The normalizations are tr$\{H_i H_j\} = \delta_{ij}/2$, tr$\{E_k E_l^\dagger\} = \delta_{kl}/2$].

Exercise 2: Consider generators of SO(3) (cf. Exercise 2.3),

$$X^1 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}, \quad X^2 = \frac{1}{2}\begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \quad X^3 = \frac{1}{2}\begin{pmatrix} 0 & i & 0 \\ -i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Find a basis in which X^1 is diagonal. What are the corresponding weight vectors?

Exercise 3: Consider now the generators

$$H = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad Y^1 = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad Y^2 = \frac{i}{2\sqrt{2}}\begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}.$$

(a) Find the linear combinations $E = c_1 Y^1 + c_2 Y^2$ with $c_i \in \mathbb{C}$, so that $[H, E] = \alpha E$. What is the corresponding root α?

(b) Show $[H, E^\dagger] = -\alpha E^\dagger$.

(c) What is the full list of roots?

Exercise 4: Apart from continuous groups, discrete groups are also important. Let S_N by the group of all permutations of N elements.

(a) Construct the multiplication table of S_3.
[*Hint*: the elements are $\{(123), (132), (213), (231), (312), (321)\}$.]

(b) Is S_3 Abelian or non-Abelian?

(c) Does S_3 have subgroups?

(d) Show that $\mathscr{D}_{(1)}[(ijk)] \equiv 1$, $\mathscr{D}_{(2)}[(ijk)] \equiv \epsilon_{ijk}$ are viable one-dimensional representations.