Standard Model	**Sheet 13**	20.05.2019

Exercise 1: Proton decay in Grand Unified Theories. In typical models of Grand Unification, a proton can decay as

$$p^+ \to e^+ \pi^0 \; .$$

On the parton level, this decay is analogous to the weak decays within the Standard Model, with $W^\pm$-exchange replaced by X, Y-boson exchange. Let M_X be the mass of the X, Y bosons, and let us assume that the corresponding gauge coupling agrees with the weak interaction one: $g \sim g_w$. Starting from the experimental lower limit of the proton lifetime, $\tau_p > 2 \times 10^{29}$ years, derive an order-of-magnitude lower bound for M_X.

Exercise 2: Unification with extra dimensions. Already before the weak and strong interactions were known, there were efforts to unify the forces known at the time, i.e. electromagnetism and gravity. One of the first proposals (G. Nordström 1914) considered the Maxwell equations as well as a competitor of Einstein's General Relativity, in which the graviton is a spin-0 rather than a spin-2 particle.

(a) Write down the Maxwell equations in $1 + 4$ dimensions.

(b) Let us write the gauge potential as

$$A_\mu = (A_0, A_1, A_2, A_3, A_4) \; , \quad \text{with} \quad A_4 =: \Phi \; .$$

Show that if the A_μ are independent of the coordinate x_4, then one obtains the Maxwell equations in $1+3$ dimensions (i.e. electromagnetism) together with an additional equation for Φ (i.e. graviton).

(c) Can you suggest a justification for the assumption that A_μ be independent of x_4?

Exercise 3: Different "symmetry breaking" patterns. If a scalar field in the adjoint rather than in the fundamental representation is used for implementing the Higgs mechanism, then there are several possible patterns for the resulting (perturbative) mass spectrum of the gauge bosons. Let us study this with the example of an SU(3) gauge theory.

(a) Taking a potential

$$V(\Xi) = -\frac{\mu^2}{2} \mathrm{Tr}\,[\Xi^2] + \frac{\lambda}{4} (\mathrm{Tr}\,[\Xi^2])^2 \; , \quad \mu^2, \lambda > 0 \; ,$$

where Ξ is a traceless and Hermitean 3×3 matrix, find the various minima. [Hint: It can be assumed that after a global transformation, Ξ is diagonal. Subsequently it can be parametrized as

$$\Xi = \begin{pmatrix} q+p & 0 & 0 \\ 0 & q-p & 0 \\ 0 & 0 & -2q \end{pmatrix} \; , \quad p, q \in \mathbb{R} \; .]$$

(b) Show that if $p = 0$ or $p = \pm 3q$, then the tree-level theory displays an SU(2)×U(1) symmetry (with 4 massless gauge bosons), whereas otherwise only a smaller U(1)×U(1) symmetry is manifest (with 2 massless gauge bosons).