Standard Model	**Sheet 11**	06.05.2019

Exercise 1: How to see experimentally the number of colours?

(a) In Exercise 7.3, the hypercharge assignments of u and d-type quarks were given. Starting from the known form of the $\mathrm{SU_L(2)\times U_Y(1)}$ covariant derivative, determine the electric charges of both quark types in units of e. [$Q_u = +\frac{2}{3}\,, Q_d = -\frac{1}{3}\,.$]

(b) In Exercises 3.3-4, the cross section for $e^-e^+ \to \mu^-\mu^+$ scattering was given. Make use of this result in order to estimate

$$R(E) \equiv \frac{\sigma(e^-e^+ \to \text{hadrons})}{\sigma(e^-e^+ \to \mu^-\mu^+)}\,,$$

where $E = \sqrt{s}$, for 1 GeV $\le E \le$ 10 GeV. [$R \approx N_\mathrm{c} \sum_{2m_\mathrm{f} \le E} Q_\mathrm{f}^2$.] (An experimental measurement of this ratio allows in principle to determine N_c, although we should note that Q_u, Q_d also change if $N_\mathrm{c} \ne 3$, as will be discussed in Exercise 12.3.)

Exercise 2: Basics of isospin conservation.

(a) If quarks are identified as the isospin states $u = |\frac{1}{2}\,, +\frac{1}{2}\,\rangle$, $d = |\frac{1}{2}\,, -\frac{1}{2}\,\rangle$, then three-quark states can have either $I = \frac{1}{2}$ or $I = \frac{3}{2}$. Suggest particles for these states, and check that those corresponding to higher-dimensional representations have heavier masses.

(b) Pions are identified as the isospin states $\pi^+ = |1, 1\rangle$, $\pi^0 = |1, 0\rangle$, $\pi^- = |1, -1\rangle$, nucleons as $p = |\frac{1}{2}\,, +\frac{1}{2}\,\rangle$, $n = |\frac{1}{2}\,, -\frac{1}{2}\,\rangle$. Considering elastic pion-nucleon scattering, there are six possible processes:

$$\begin{aligned} &\pi^+ + p \to \pi^+ + p\,, \qquad &&\pi^0 + p \to \pi^0 + p\,, \qquad &&\pi^- + p \to \pi^- + p\,, \\ &\pi^+ + n \to \pi^+ + n\,, \qquad &&\pi^0 + n \to \pi^0 + n\,, \qquad &&\pi^- + n \to \pi^- + n\,. \end{aligned}$$

How many independent amplitudes are there in these scatterings if isospin symmetry is assumed exact? Can you express all amplitudes in terms of the independent ones?

Exercise 3: Basics of the parton model. In the lecture the variables $Q_\mathrm{E}^2 := -Q^2$ and $x = Q_\mathrm{E}^2/(2Q \cdot P)$ were defined for deep inelastic scattering; here P is the proton and Q the photon four-momentum. The energy can be assumed to be so large that effectively $m_e = 0$.

(a) Show that $0 \le x \le 1$.

The parton distribution functions, $f_i(x)$, satisfy "sum rules", for instance $\int_0^1 \mathrm{d}x\, x \sum_i f_i(x) = 1$ as discussed in the lecture. Let us assume that proton structure can de described by the distributions $u_v(x)$, $d_v(x)$, $s(x)$, $\bar{s}(x)$, $g(x)$. Which sum rules capture the facts that

(b) proton has electric charge $Q = +1$?

(c) proton has no net strangeness?