Standard Model	**Sheet 10**	29.04.2019

Exercise 1: Yukawa-mediated decays. The Higgs boson is short-lived, and is searched for by trying to identify its decay products.

(a) If $M_H = 125$ GeV, what is the most important tree-level decay channel?

(b) Consider the decay $H \to \gamma\gamma$. This does not take place at tree level (why?), but show that there are loop diagrams allowing for the decay.

Exercise 2: CP-violating oscillations. In Exercise 4.1 two states $|K^0\rangle$, $|\bar{K}^0\rangle$ were considered, as well as their linear combinations $|K^0_\pm\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle \mp |\bar{K}^0\rangle)$, so that $\hat{C}\hat{P}|K^0_\pm\rangle = \pm|K^0_\pm\rangle$.

(a) Let H be a complex 2×2-matrix, composed of matrix elements of the Hamiltonian $\hat{H}$:

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} := \begin{pmatrix} \langle K^0|\hat{H}|K^0\rangle & \langle K^0|\hat{H}|\bar{K}^0\rangle \\ \langle \bar{K}^0|\hat{H}|K^0\rangle & \langle \bar{K}^0|\hat{H}|\bar{K}^0\rangle \end{pmatrix}.$$

Draw Feynman diagrams showing that in general $H_{12} \neq 0$.

(b) Let us assume that $H_{11} = H_{22}$ (this follows from the CPT-theorem) and that H is Hermitean. Show that if H_{12} has a non-trivial complex phase, then the mass eigenstates (i.e. eigenstates of H, conventionally denoted by $|K^0_S\rangle$, $|K^0_L\rangle$) are *not* identical to $|K^0_\pm\rangle$.

(c) Suppose that the system is created in the state $|K^0\rangle$ at time $t = 0$. Determine the probability to find the system in the state $|K^0_-\rangle$ at time t.

(Note: In reality the kaons decay; this means that H is not Hermitean but can be expressed as $H = M - \frac{i}{2}\Gamma$, where M and Γ are Hermitean. Sometimes $|K^0_L\rangle$ decays into a CP-even state; this is called *indirect* CP-violation, if it is due to the mixing of $|K^0_L\rangle$ and $|K^0_+\rangle$, and *direct* CP-violation, if the dominant component $|K^0_-\rangle$ of $|K^0_L\rangle$ decays.)

Exercise 3: Unitarity triangle. Within the Standard Model, the CKM matrix

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{\mathsf{ud}} & V_{\mathsf{us}} & V_{\mathsf{ub}} \\ V_{\mathsf{cd}} & V_{\mathsf{cs}} & V_{\mathsf{cb}} \\ V_{\mathsf{td}} & V_{\mathsf{ts}} & V_{\mathsf{tb}} \end{pmatrix}$$

is unitary. If the elements V_{kl} can be measured experimentally, it could however turn out that they do not satisfy the unitarity constraint, which would then be an indication that Standard Model is incomplete; the issue is particularly relevant for CP-violation.

(a) Show that one of the constraints reads $V_{\mathsf{ud}}V^*_{\mathsf{ub}} + V_{\mathsf{cd}}V^*_{\mathsf{cb}} + V_{\mathsf{td}}V^*_{\mathsf{tb}} = 0$.

(b) Writing $\frac{V_{\mathsf{td}}V^*_{\mathsf{tb}}}{V_{\mathsf{cd}}V^*_{\mathsf{cb}}} =: \left|\frac{V_{\mathsf{td}}V^*_{\mathsf{tb}}}{V_{\mathsf{cd}}V^*_{\mathsf{cb}}}\right| e^{i\phi_1}$, $\frac{V_{\mathsf{ud}}V^*_{\mathsf{ub}}}{V_{\mathsf{td}}V^*_{\mathsf{tb}}} =: \left|\frac{V_{\mathsf{ud}}V^*_{\mathsf{ub}}}{V_{\mathsf{td}}V^*_{\mathsf{tb}}}\right| e^{i\phi_2}$, $\frac{V_{\mathsf{cd}}V^*_{\mathsf{cb}}}{V_{\mathsf{ud}}V^*_{\mathsf{ub}}} =: \left|\frac{V_{\mathsf{cd}}V^*_{\mathsf{cb}}}{V_{\mathsf{ud}}V^*_{\mathsf{ub}}}\right| e^{i\phi_3}$, verify that the following triangle can be constructed ($\bar{\phi}_i \equiv \pi - \phi_i$):