Standard Model	**Sheet 9**	15.04.2019

Exercise 1: Counting degrees of freedom. A spin-s particle has $2s+1$ different polarization states; however, a *massless* spin-1 particle, such as a photon, only has 2 states. Consider the Higgs doublet as well as the $\mathrm{SU_L}(2)$ and $\mathrm{U_Y}(1)$ gauge bosons.

(a) Count the number of physical degrees of freedom if there is no symmetry breaking.

(b) Repeat the exercise in the presence of a Higgs vacuum expectation value.

Exercise 2: Goldstone's theorem. In the case of a spontaneous breaking of a *global* symmetry, the spectrum contains a number of massless modes, called Goldstone bosons. Let us verify this in the case of a theory of N real scalar fields ϕ_k, $k = 1, ..., N$, invariant under $\mathrm{O}(N)$ symmetry. Let ϕ denote a vector which has the ϕ_k as its components; $V(\phi)$ the potential; and ϕ_0 its minimum. Taylor-expanding around the minimum,

$$V(\phi) = V(\phi_0) + \frac{1}{2}\sum_{k,l}(\phi-\phi_0)_k(\phi-\phi_0)_l \left.\frac{\partial^2 V(\phi)}{\partial\phi_k\partial\phi_l}\right|_{\phi=\phi_0} + \dots \ ,$$

we identify a matrix $M_{kl} := \partial_{\phi_k}\partial_{\phi_l}V(\phi)|_{\phi=\phi_0}$ whose eigenvalues represent squared masses.

(a) Consider a field transformation

$$\phi_k \to \phi'_k = \phi_k + \epsilon\,\Delta_k(\phi)$$

which leaves the potential invariant: $V(\phi) = V(\phi')$. Expanding to first order in ϵ and differentiating subsequently with respect to ϕ_l, derive the following equation:

$$\sum_k \Delta_k(\phi_0)\, M_{kl} = 0 \ .$$

(b) Show that each direction in the field space which rotates ϕ away from the ground state ϕ_0 but leaves the potential invariant, corresponds to a massless excitation.

(c) Show that $\mathrm{O}(N)$ transformations are parametrized by in total $N(N-1)/2$ parameters.

(d) Arguing that after the symmetry breaking, there still remains a group $\mathrm{O}(N-1)$ of transformations which do *not* rotate ϕ away from ϕ_0, show that the spontaneous breaking of $\mathrm{O}(N)$ leads to the existence of $N-1$ Goldstone bosons.

Exercise 3: Basic phenomenology. Within the Standard Model the parameters of the Higgs potential, μ^2 and λ, are free, but there are ways in which to reduce this freedom.

(a) Recalling that g_w and M_W are known, write a tree-level relation between μ^2 and λ.

(b) Express the scalar self-coupling λ as a function of g_w, M_W and M_H.

(c) Since the Standard Model is phenomenologically relatively successful within tree-level relations, we might expect the Higgs sector to be "weakly coupled". What do you obtain for λ for the physical value of M_H?

(d) In many extensions of the Standard Model, the parameter λ is no longer free, but a function of other parameters. For instance, within the so-called Minimal Supersymmetric Standard Model, it can be argued that $\lambda \lesssim g_w^2/2$. Which is the allowed range of M_H?