Standard Model	**Sheet 6**	25.03.2019

Exercise 1: Basic phenomenology.

(a) In the lecture we obtained the relation

$$g_w^2 = 4\sqrt{2} M_W^2 G_F \; .$$

Starting from the known M_W and G_F, determine the numerical value of the weak fine-structure constant $\alpha_w := g_w^2/(4\pi)$, and compare with α_{em}. Why is it that weak interactions are "weak" compared with electromagnetic interactions?

(b) Other relations (to be obtained in sec. 3.5) are

$$e = g_w \sin\theta_w \; , \quad \sin\theta_w = \frac{g_Y}{\sqrt{g_w^2 + g_Y^2}} \; , \quad \frac{M_Z}{M_W} = \frac{\sqrt{g_w^2 + g_Y^2}}{g_w} \; ,$$

where $\alpha_{em} = e^2/(4\pi)$. Use these to obtain a "tree-level" prediction for M_Z/M_W, by eliminating the unknowns g_Y, $\sin\theta_w$. How well does this compare with experiment?

Exercise 2: Ultraviolet divergences.

(a) Consider the integral

$$A(m^2, \Lambda) \equiv \int_{|\mathbf{r}|<\Lambda} \frac{\mathrm{d}^3\mathbf{r}}{(2\pi)^3} \int_{-\infty}^{+\infty} \frac{\mathrm{d}r_0}{2\pi} \, \frac{1}{R^2 - m^2 + i0^+} \; .$$

How does $A(m^2, \Lambda)$ behave for $\Lambda^2 \gg m^2$? (Keep all terms that do not vanish.)

(b) Consider then

$$B(m^2, Q^2, \Lambda) \equiv \int_{|\mathbf{r}|<\Lambda} \frac{\mathrm{d}^3\mathbf{r}}{(2\pi)^3} \int_{-\infty}^{+\infty} \frac{\mathrm{d}r_0}{2\pi} \, \frac{1}{[R^2 - m^2 + i0^+]\,[(Q+R)^2 - m^2 + i0^+]} \; .$$

How does $B(m^2, Q^2, \Lambda)$ behave for $\Lambda^2 \gg m^2, Q^2$? [Hint: Carry out a Taylor-expansion in Q^2; compute the leading term explicitly; and argue that all other terms of the series remain finite for $\Lambda \to \infty$.]

Exercise 3: Loss of predictive power. In the script the problems of "unitarity violation" and "infinitely large loop corrections" were treated separately, but in a sense the two issues are also related to each other. Let us stick with the Fermi model and *assume* that infinities can be hidden through the introduction of suitable "counterterms", and that therefore loop corrections can be computed to any quantity, for instance to $\sigma(s)$.

(a) Draw representative Feynman diagrams within the Fermi model leading to loop corrections to the cross section for the reaction $\bar{\nu}_e e^- \to \bar{\nu}_e e^-$.

(b) Through dimensional analysis (without explicit computation), argue how the corresponding corrections scale as a function of s, for $\sqrt{s} \gg m_e$.

(c) Show that even if all corrections were ultraviolet finite, then the perturbative series is nevertheless likely to lose its convergence when $s \geq G_F^{-1}$. [We say that the theory becomes "strongly coupled" at this scale.]