Standard Model	**Sheet 5**	18.03.2019

Exercise 1: Helicity. Consider an on-shell spinor like in Exercise 3.1, but now with a massless particle: $u(\vec{p},s) = \frac{1}{\sqrt{p}}\not{P}\,\xi_s$, $\xi_+ := (1\,0\,0\,0)^T$, $\xi_- := (0\,1\,0\,0)^T$. (We use the standard representation here:

$$\gamma^0 = \begin{pmatrix} \mathbb{1}_{2\times 2} & 0 \\ 0 & -\mathbb{1}_{2\times 2} \end{pmatrix}\,, \quad \gamma^k = \begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix}\,,$$

where σ^k are the Pauli matrices.) For a particle moving in the z-direction, with $\vec{p} = (0,0,p)$, its "helicity" or spin state, can be measured with the operator

$$\Sigma_z = \frac{1}{2}\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}\,.$$

Show that $2\Sigma_z\, u(\vec{p},s) = s\, u(\vec{p},s) = \gamma^5\, u(\vec{p},s)$, i.e. that the massless spinor is a simultaneous eigenstate both of helicity and chirality (the latter was defined in Exercise 4.3), and that the projection u_{L} carries negative helicity.

Exercise 2: Non-perturbative amplitudes. Consider the decay $\pi^+ \to \mu^+\nu_\mu$, with $\pi^+ = u\bar{d}$.

(a) Starting from the V-A Fermi model, write down the relevant part of the interaction Lagrangian. [$\mathscr{L}_I^{\text{V-A}} = -\frac{G_F}{\sqrt{2}}\cos\theta_c\, \bar{d}\gamma^\alpha(1-\gamma^5)u\; \bar{\nu}_\mu\gamma_\alpha(1-\gamma^5)\mu$.]

(b) The hadronic part of decay matrix element can be parametrized as

$$\langle 0|\bar{d}\gamma^\alpha(1-\gamma^5)u|\pi^+(\vec{k})\rangle = i\sqrt{2}F_\pi K^\alpha\,,$$

where F_π is the "pion decay constant". Write down the corresponding amplitude. [$\mathscr{M} = \eta\, G_F F_\pi \cos\theta_c K^\alpha \bar{u}_{\nu_\mu}(\vec{p}_1,t_1)\gamma_\alpha(1-\gamma^5)v_\mu(\vec{p}_2,t_2)$, $\eta = i^n$.]

(c) By making use of the Dirac equation and four-momentum conservation, show that

$$K^\alpha \bar{u}_{\nu_\mu}\gamma_\alpha(1-\gamma^5)v_\mu = -m_\mu \bar{u}_{\nu_\mu}(1+\gamma^5)v_\mu\,.$$

(d) Summing over the spins of the final state particles, show finally that

$$\sum|\mathscr{M}|^2 = 4G_F^2F_\pi^2\cos^2\theta_c\, m_\mu^2(m_\pi^2 - m_\mu^2)\,.$$

Exercise 3: GIM mechanism. Consider the decay $K^0 \to \mu^+\mu^-$, where $K^0 = d\bar{s}$. This requires a transition $d \to u \to s$ or $d \to c \to s$, so that s and $\bar{s}$ can annihilate, leaving over the $\mu^-\mu^+$ pair. Starting from the V-A Fermi model:

(a) Draw the corresponding Feynman diagrams.

(b) Show that the two channels to a good approximation cancel against each other.

(c) Why is the cancellation not exact?

(This cancellation is referred to as the GIM mechanism [Glashow, Iliopoulos, Maiani 1970]: the fourth quark c was introduced, before its experimental discovery, in order to explain the fact that the decay rate $\Gamma(K^0 \to \mu^+\mu^-)$ was observed to be very slow.)