Standard Model	**Sheet 4**	11.03.2019

Exercise 1: Eigenstates of C and P. Neutral kaons can be represented by "currents",

$$K^0 = Z\,\bar{\psi}_d\, i\gamma^5\psi_s\ , \quad \bar{K}^0 = Z\,\bar{\psi}_s i\gamma^5\psi_d\ ,$$

where Z is some constant.

(a) Show that in the transformations P and C,

$$K^0 \xrightarrow{P} -K^0\ , \quad \bar{K}^0 \xrightarrow{P} -\bar{K}^0\ , \quad K^0 \xrightarrow{C} \bar{K}^0\ , \quad \bar{K}^0 \xrightarrow{C} K^0\ .$$

(b) If we consider the corresponding quantum-mechanical states, then

$$\hat{P}|K^0\rangle = -|K^0\rangle\ , \quad \hat{P}|\bar{K}^0\rangle = -|\bar{K}^0\rangle\ , \quad \hat{C}|K^0\rangle = |\bar{K}^0\rangle\ , \quad \hat{C}|\bar{K}^0\rangle = |K^0\rangle\ .$$

Construct linear combinations of $|K^0\rangle$, $|\bar{K}^0\rangle$ which are eigenstates of $\hat{C}\hat{P}$.

Exercise 2: Charge conjugation. The charge conjugation matrix should satisfy the following relations:

$$C = -C^T\ , \quad C(-\gamma^\mu)^T C^{-1} = \gamma^\mu\ .$$

Show that:

(a) The matrices $-(\gamma^\mu)^T$ satisfy the Clifford algebra and that in this sense C is a similarity transformation between different representations of the Clifford algebra.

(b) If we define $\hat{\gamma}_0 := \gamma^0$, $\hat{\gamma}_k := -i\gamma^k$, $k = 1,2,3$, which are Hermitean matrices, then the transformation can be written as $C(-\hat{\gamma}^*_\mu)C^{-1} = \hat{\gamma}_\mu$, so that C must anti-commute with any purely real $\hat{\gamma}_\mu$. [Hint: Make use of the general property $(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0$.]

(c) If C is expressed as a product of two different real $\hat{\gamma}_\mu$ matrices, so that it automatically anticommutes with them both, then also the first property $C = -C^T$ is satisfied.

(Note: In the "standard representation", $\hat{\gamma}_0$ and $\hat{\gamma}_2$ are real and $C = \hat{\gamma}_0\hat{\gamma}_2$.)

Exercise 3: Chirality. With the known matrix γ^5 ($\gamma^5 := i\gamma^0\gamma^1\gamma^2\gamma^3$; $(\gamma^5)^2 = \mathbb{1}$; $\{\gamma^5, \gamma^{\tilde{\mu}}\} = 0$, $\tilde{\mu} = 0,1,2,3$), which is defined to measure "chirality", we can define

$$\mathbb{P}_{\mathsf{L}} := \frac{\mathbb{1} - \gamma^5}{2}\ , \quad \mathbb{P}_{\mathsf{R}} := \frac{\mathbb{1} + \gamma^5}{2}\ .$$

(It is easily verified that $\mathbb{P}_{\mathsf{L,R}}$ are projection operators: $\mathbb{1} = \mathbb{P}_{\mathsf{L}} + \mathbb{P}_{\mathsf{R}}$, $\mathbb{P}_{\mathsf{L}}^2 = \mathbb{P}_{\mathsf{L}}$, $\mathbb{P}_{\mathsf{R}}^2 = \mathbb{P}_{\mathsf{R}}$, $\mathbb{P}_{\mathsf{L}}\mathbb{P}_{\mathsf{R}} = 0$.) In so-called dimensional regularization, the theory also has additional γ-matrices, with the property $[\gamma^5, \gamma^{\bar{\mu}}] = 0$, $\bar{\mu} > 3$. The index μ includes both $\tilde{\mu}$ and $\bar{\mu}$.

(a) Defining $\psi_{\mathsf{L}} \equiv \mathbb{P}_{\mathsf{L}}\psi$, $\psi_{\mathsf{R}} \equiv \mathbb{P}_{\mathsf{R}}\psi$, show that $\gamma^5\psi_{\mathsf{L}} = -\psi_{\mathsf{L}}$, $\gamma^5\psi_{\mathsf{R}} = \psi_{\mathsf{R}}$, i.e. $\psi_{\mathsf{L,R}}$ are eigenstates of chirality.

(b) Show that in the massless limit, $\mathscr{L}_\psi \equiv \bar{\psi}\, i\gamma^\mu D_\mu\, \psi$ can be split into a sum of three terms: one for ψ_{L}, one for ψ_{R}, and a third mixing $\psi_{\mathsf{L,R}}$ but only involving the "extra" indices $\bar{\mu}$.